

Generative Model Variants

-- *GLO, IMLE, GLANN*

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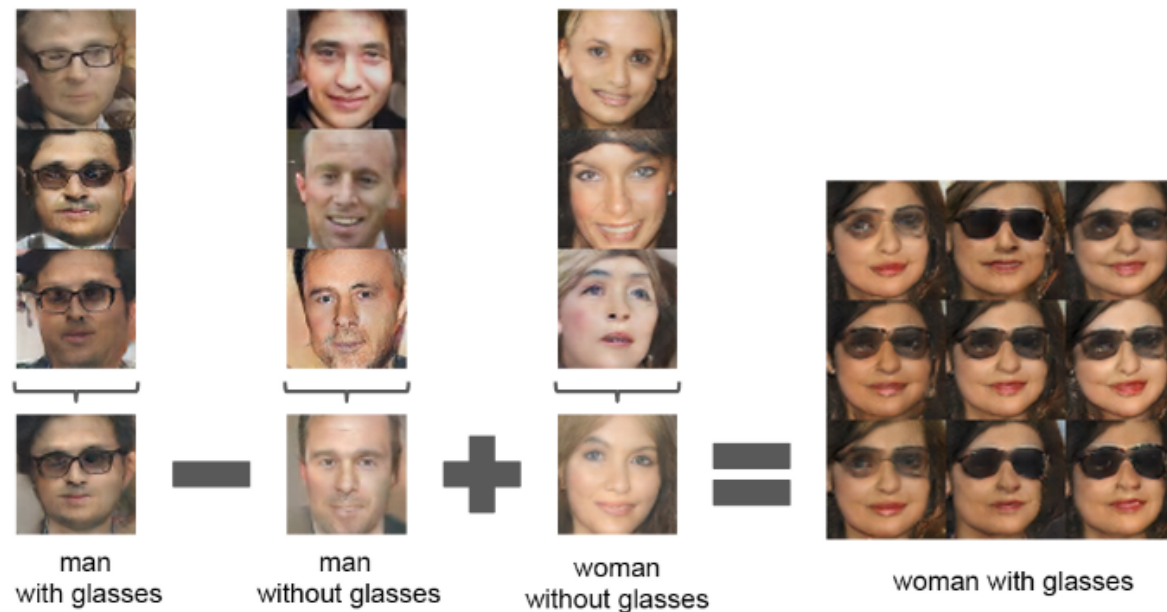
Generative Model Variants -- GLO, IMLE, GLANN

- Generative Latent Optimisation, GLO
- Implicit Maximum Likelihood Estimation, IMLE
- Generative Latent Nearest Neighbors, GLANN
- Discussion

Recap: DCGAN

- **Properties of DCGAN**

- Generation from noise: synthesis new images from data distribution
- Interpolation: translate linear interpolation in noise space to semantic interpolation in image
- Linear operation: linear arithmetic in noise space



Recap: DCGAN

- **Properties of DCGAN**

- Mode collapse: generator may model only a few localised regions of data distribution
- Adversarial training: unstable for training, sensitive to initialisation, architecture and hyper parameters

- **GLO**
 - **IMLE**
 - **GLANN**
- } Aim at solving major problems in GANs to develop NEW generative models.

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Generative Latent Optimisation (GLO)

“Success of GANs comes from:

- Inductive bias of deep convolutional networks
- The adversarial training protocol ”

GLO: aims at preserving good properties of GAN, without adversarial training and mode collapse

Approach:

- Image space \mathcal{X} , noise space $\mathcal{Z} \subset \mathbb{R}^d$. Training set of images $\{x_1, \dots, x_N\}$.
- For each image x_i , take a random noise $z_i \in \mathcal{Z}$.
- Construct all the pairs: $\{(z_i, x_i)\}_{i=1}^N$
- Target is to learn the mapping $g_\theta: \mathcal{Z} \rightarrow \mathcal{X}$, where each training image x_i is mapped from its noise z_i

Generative Latent Optimisation (GLO)

Approach:

- Parameterize a generator $g_\theta: \mathcal{Z} \rightarrow \mathcal{X}$ using CNN
- Initialize $\theta, \{z_i\}$
- Training objective:

$$\min_{\theta} \sum_{i=1}^N [\min_{z_i \in \mathcal{Z}} \ell(g_\theta(z_i), x_i)]$$

Notice:

- Noise z_i are learnable, jointly optimised with θ , to form the structure of the noise space.
- It is like a “non-parameterized autoencoder”.

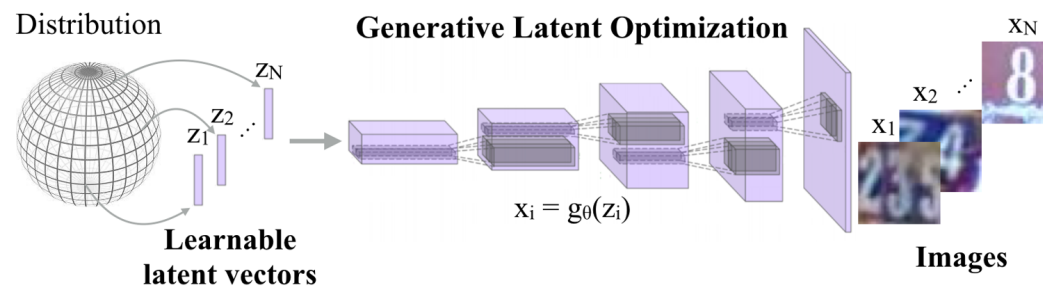
Generative Latent Optimisation (GLO)

Noise space

- A common choice of \mathcal{Z} in the GANs literature is unit Gaussian distribution from \mathbb{R}^d

$$z \sim \mathcal{N}(0, I)$$

- z drawn from unit Gaussian distribution is unlikely to land far outside the sphere of radius \sqrt{d} , i.e. with high probability, $\|z\|_2 \in (\sqrt{d} - \varepsilon, \sqrt{d} + \varepsilon)$.
- Instead of using Gaussian, GLO use the unit sphere as the noise space.



Generative Latent Optimisation (GLO)

Noise space

Think: why not using a Gaussian distribution?

Because in the optimisation objective, the noise vectors need to be optimised. It is easy to constrain them into a sphere, by optimising and projecting them back to the sphere for each step.

Generative Latent Optimisation (GLO)

Loss function

- Simple choice: squared error

$$\ell_2(x, x') = \|x - x'\|_2^2$$

- Leads to blurry reconstructions of natural images (moving to the average)
- GANs generate sharp image, because they use a CNN (discriminator) as the loss function, whose early layers focus on edges.
- GLO uses Laplacian pyramid loss for sharp reconstruction.

Generative Latent Optimisation (GLO)

Loss function

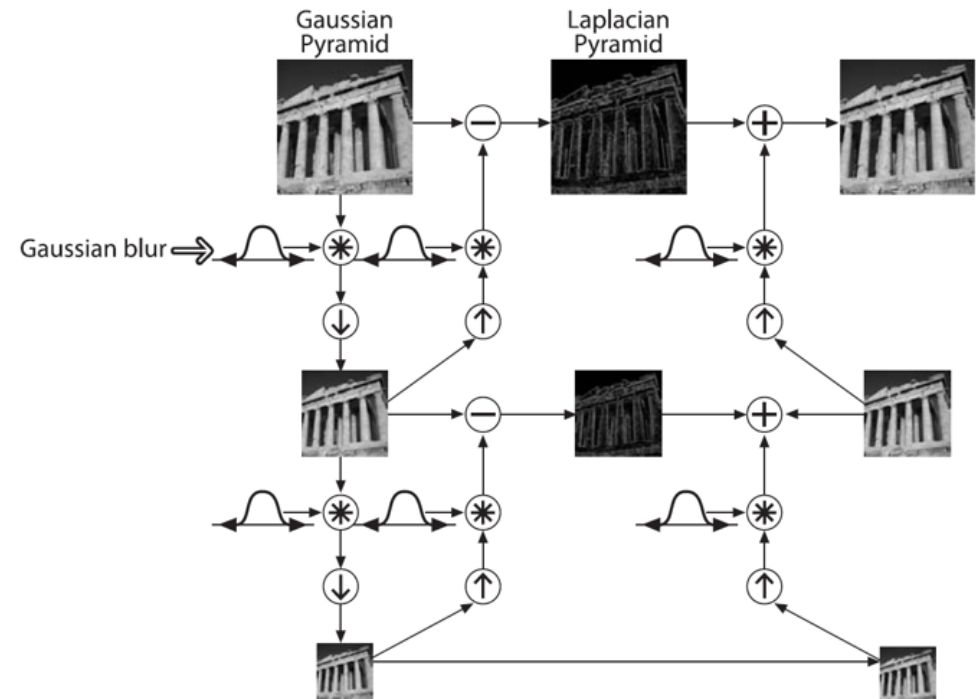
- Laplacian pyramid loss:

$$Lap(x, x') = \sum_j 2^{2j} |L^j(x) - L^j(x')|_1$$

- L^j is the j^{th} level residual in the image's Laplacian pyramid, which is the residual between the j^{th} level image and the upsampled $j + 1^{th}$ level, reflecting the details lost during upsampling.

$$L^j(x) = G^j(x) - Upsample(G^{j+1}(x))$$

G^j is the j^{th} downsampled image in the Gaussian pyramid.

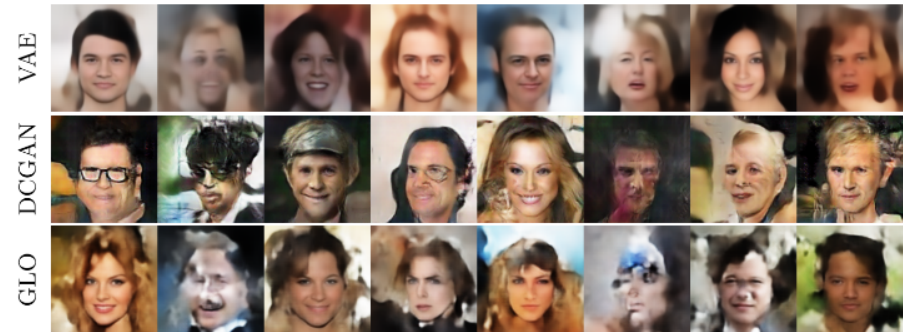
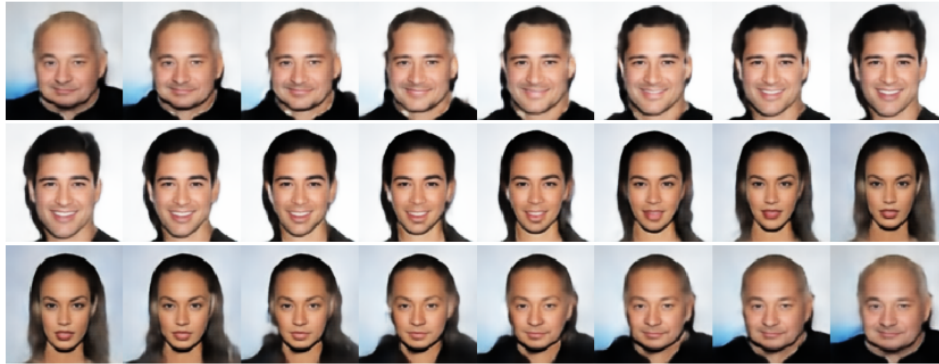


Generative Latent Optimisation (GLO)

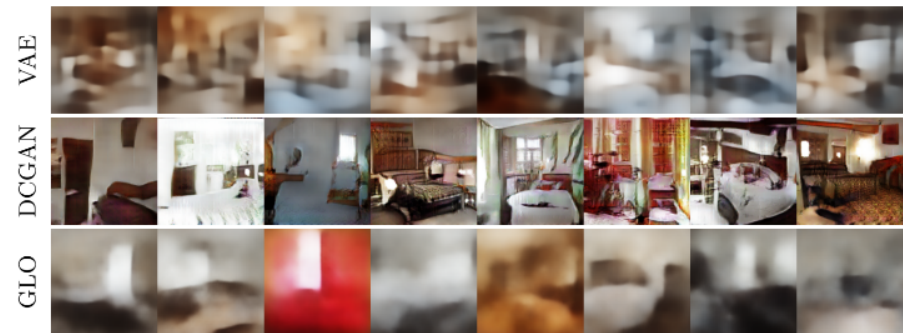
Discussion

- Generation
 - Distribution of noise on the unit sphere is unknown
 - Need to fit another (simple) model to map a known distribution to the noise distribution
 - E.g. fit a full-covariance Gaussian distribution
- Properties of latent interpolation & linear operation are preserved.
- Mode Collapse
 - Issue of mode collapse is addressed, because all training samples are embedded in the noise space.

Generative Latent Optimisation (GLO)



(d) CelebA-128



(f) LSUN-128

- Nice result on interpolation for images in the training set.
- GLO cannot outperform DCGAN on image generation.
 - Noise space for GLO is more like a feature embedding space, not for sampling.

- Generative Latent Optimisation, GLO
- **Implicit Maximum Likelihood Estimation, IMLE**
- Generative Latent Nearest Neighbors, GLANN
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Implicit Maximum Likelihood Estimation (IMLE)

Recall

- Generative models aim at model the data distribution $x \sim p_{data}$

- A common approach is to
 - sample from a known distribution

$$z \sim q(z)$$

- then use a parametric function to map the known distribution to the data distribution

$$x = G_{\theta}(z)$$

- Denote the learned distribution $x \sim p_{\theta}$

Implicit Maximum Likelihood Estimation (IMLE)

Recall

- Maximum likelihood estimation (MLE)
 - Maximise the probability of p_θ to generate the data sampled from p_{data}

- MLE is equivalent to minimising KL divergence between two distributions

$$\min_{\theta} D_{KL}(p_{data} || p_{\theta}) = \min_{\theta} \int_x p_{data} \log\left(\frac{p_{data}}{p_{\theta}}\right) dx$$

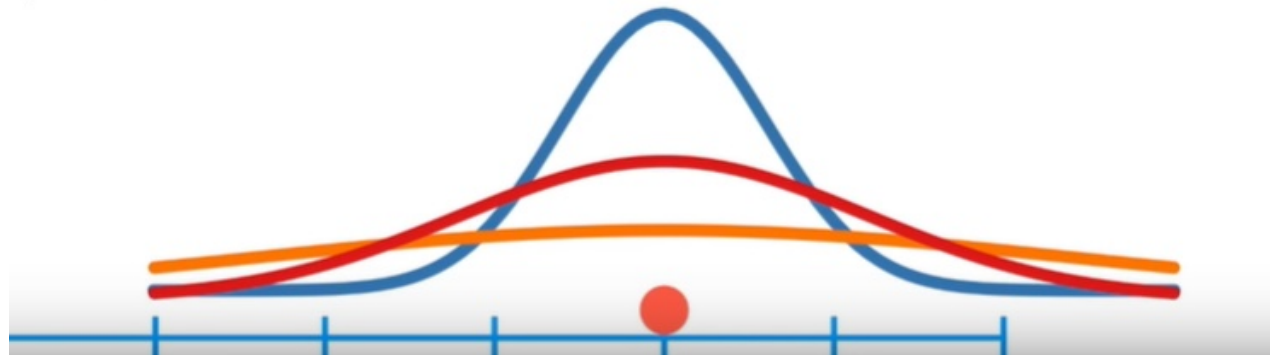
- However, the probability density function of p_θ is always intractable

$$p_{\theta}(x) = \frac{\partial}{\partial x_1} \dots \frac{\partial}{\partial x_d} \int_{\{z | \forall i, (G_{\theta}(z))_i \leq x_i\}} q(z) dz$$

Implicit Maximum Likelihood Estimation (IMLE)

Intuition

- MLE should assign high probability density to each of the samples



- We can adjust the parameter θ so that samples drawn from p_θ are close to samples drawn from p_{data}

Implicit Maximum Likelihood Estimation (IMLE)

Algorithm

Require: The dataset $D = \{\mathbf{x}_i\}_{i=1}^n$ and a sampling mechanism for the implicit model P_θ
Initialize θ to a random vector
for $k = 1$ **to** K **do**
 Draw i.i.d. samples $\tilde{\mathbf{x}}_1^\theta, \dots, \tilde{\mathbf{x}}_m^\theta$ from P_θ
 Pick a random batch $S \subseteq \{1, \dots, n\}$
 $\sigma(i) \leftarrow \arg \min_j \|\mathbf{x}_i - \tilde{\mathbf{x}}_j^\theta\|_2^2 \quad \forall i \in S$
 for $l = 1$ **to** L **do**
 Pick a random mini-batch $\tilde{S} \subseteq S$
 $\theta \leftarrow \theta - \eta \nabla_\theta \left(\frac{n}{|\tilde{S}|} \sum_{i \in \tilde{S}} \|\mathbf{x}_i - \tilde{\mathbf{x}}_{\sigma(i)}^\theta\|_2^2 \right)$
 end for
end for
return θ

Implicit Maximum Likelihood Estimation (IMLE)

Algorithm analysis

- Mode collapse is addressed, because the training process encourages the generated samples to be close to all the samples in training data.
- The training objective is a simple l_2 loss. It is easy to optimise.
- Distance measurement to compute the nearest neighbor and to optimise is critical. L_2 distance used in paper leads to generating blur images.

Implicit Maximum Likelihood Estimation (IMLE)

Algorithm analysis

- L_2 distance used in paper leads to blur results.



IMLE trained on cifar-10.

Implicit Maximum Likelihood Estimation (IMLE)

Theoretical analysis

Why does this simple algorithm work?

- IMLE has a theoretical proof, showing the algorithm is equivalent to MLE.

Recap:

- $z \sim q(z)$
- $x = G_\theta(z)$
- $p_\theta(x) = \frac{\partial}{\partial x_1} \cdots \frac{\partial}{\partial x_d} \int_{\{z | \forall i, (G_\theta(z))_i \leq x_i\}} q(z) dz$
- MLE Objective: $\min_{\theta} D_{KL}(p_{data} || p_\theta)$

Implicit Maximum Likelihood Estimation (IMLE)

Theoretical Analysis

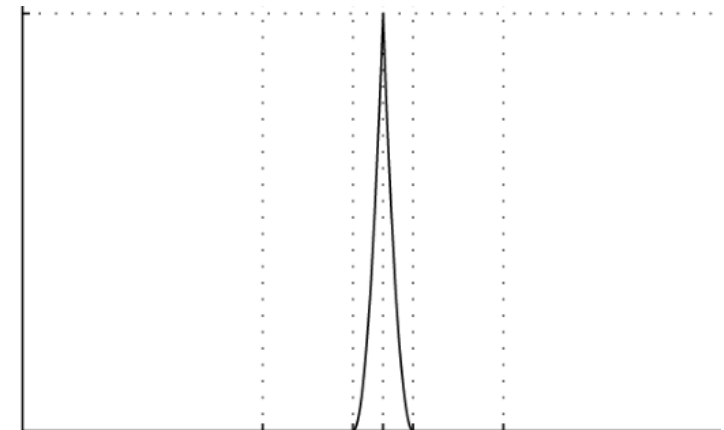
Lemma : $p_{\theta}(x) = \int \delta(x - G(z))q(z)dz$, where δ is the Dirac function.

Dirac δ function is a generalised function satisfies:

$$\delta(x) = \begin{cases} 0, & x \neq 0 \\ \infty, & x = 0 \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(x) = 1$$

Property: for any function f , $\int_{-\infty}^{+\infty} f(x + y)\delta(x) dx = f(y)$



Implicit Maximum Likelihood Estimation (IMLE)

Theoretical Analysis

Lemma : $p_{\theta}(x) = \int \delta(x - G(z))q(z)dz$, where δ is the Dirac function.

Proof:

$$\begin{aligned} & \int_{-\infty}^x dt \int \delta(t - G(z))q(z)dz \\ &= \int \left\{ q(z)dz \int_{-\infty}^x \delta(t - G(z)) dt \right\} \\ &= \int \left\{ q(z)dz \mathbf{I} \left[\forall i, (G_{\theta}(z))_i < (x)_i \right] \right\} \\ &= \int_{\{z | \forall i, (G_{\theta}(z))_i \leq x_i\}} q(z)dz \\ &= \int_{-\infty}^x p_{\theta}(x)dx \end{aligned}$$

Implicit Maximum Likelihood Estimation (IMLE)

Theoretical Analysis

$$p_{\theta}(x) = \int \delta(x - G(z))q(z)dz = \mathbb{E}_{z \sim q(z)}[\delta(x - G(z))]$$

Dirac function is also a limit of isotropic Gaussian distribution:

$$\delta(x) = \lim_{\sigma \rightarrow 0} \frac{1}{(2\pi\sigma^2)^{d/2}} \exp\left(-\frac{\|x\|^2}{2\sigma^2}\right)$$

Then,

$$p_{\theta}(x) = \lim_{\sigma \rightarrow 0} \mathbb{E}_{z \sim q(z)} \left[\frac{1}{(2\pi\sigma^2)^{d/2}} \exp\left(-\frac{\|x - G(z)\|^2}{2\sigma^2}\right) \right]$$

Implicit Maximum Likelihood Estimation (IMLE)

Theoretical Analysis

MLE Objective:

$$\begin{aligned}
 & \underset{\theta}{\text{minimise}} D_{KL}(p_{data} || p_{\theta}) \\
 \Leftrightarrow & \underset{\theta}{\text{minimise}} \int_x p_{data} \log\left(\frac{p_{data}}{p_{\theta}}\right) dx \\
 \Leftrightarrow & \underset{\theta}{\text{minimise}} \mathbb{E}_{x \sim p_{data}(x)} [-\log(p_{\theta})] \\
 \Leftrightarrow & \underset{\theta}{\text{minimise}} \mathbb{E}_{x \sim p_{data}(x)} \left[-\log \left\{ \lim_{\sigma \rightarrow 0} \mathbb{E}_{z \sim q(z)} \left[\frac{1}{(2\pi\sigma^2)^{\frac{d}{2}}} \exp\left(-\frac{\|x - G(z)\|^2}{2\sigma^2}\right) \right] \right\} \right] \\
 \Leftrightarrow & \underset{\theta}{\text{minimise}} \lim_{\sigma \rightarrow 0} \mathbb{E}_{x \sim p_{data}(x)} \left[-\log \left\{ \mathbb{E}_{z \sim q(z)} \left[\exp\left(-\frac{\|x - G(z)\|^2}{2\sigma^2}\right) \right] \right\} \right]
 \end{aligned}$$

Implicit Maximum Likelihood Estimation (IMLE)

Theoretical Analysis

MLE Objective

- In practice, we sample the data from distribution to approximate the expectation
- Suppose x_1, x_2, \dots, x_M are drawn from p_{data} (training data); z_1, z_2, \dots, z_N are drawn from $q(z)$

$$\begin{aligned} & \underset{\theta}{\text{minimise}} \lim_{\sigma \rightarrow 0} \mathbb{E}_{x \sim p_{data}(x)} \left[-\log \left\{ \mathbb{E}_{z \sim q(z)} \left[\exp \left(-\frac{\|x - G(z)\|^2}{2\sigma^2} \right) \right] \right\} \right] \\ & \Leftrightarrow \underset{\theta}{\text{minimise}} \lim_{\sigma \rightarrow 0} -\frac{1}{M} \sum_{i=1}^M \log \left\{ \sum_{j=1}^N \exp \left(-\frac{\|x_i - G(z_j)\|^2}{2\sigma^2} \right) \right\} \end{aligned}$$

Implicit Maximum Likelihood Estimation (IMLE)

Theoretical Analysis

MLE Objective

$$\underset{\theta}{\text{minimise}} \lim_{\sigma \rightarrow 0} -\frac{1}{M} \sum_{i=1}^M \log \left\{ \sum_{j=1}^N \exp \left(-\frac{\|x_i - G(z_j)\|^2}{2\sigma^2} \right) \right\}$$

$$\Leftrightarrow \underset{\theta}{\text{minimise}} -\sum_{i=1}^M \max_{j=1}^N \left\{ -\|x_i - G(z_j)\|^2 \right\}$$

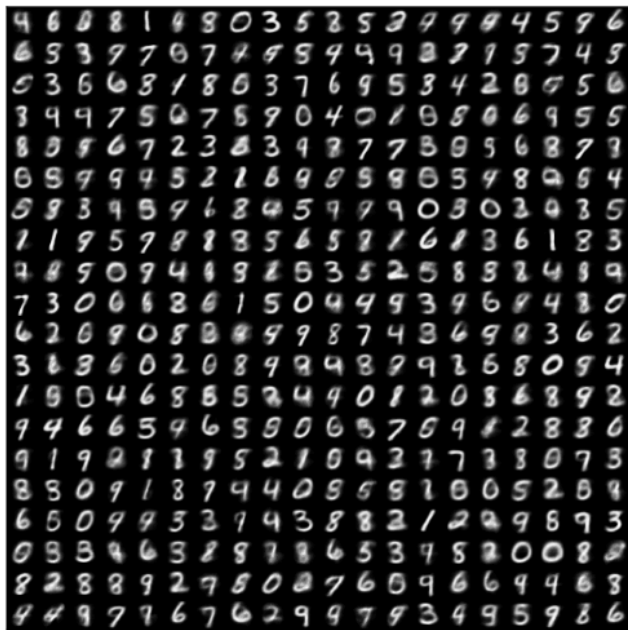
$$\Leftrightarrow \underset{\theta}{\text{minimise}} \sum_{i=1}^M \min_{j=1}^N \left\{ \|x_i - G(z_j)\|^2 \right\}$$

Minimise the nearest neighbor distance

Implicit Maximum Likelihood Estimation (IMLE)

Results

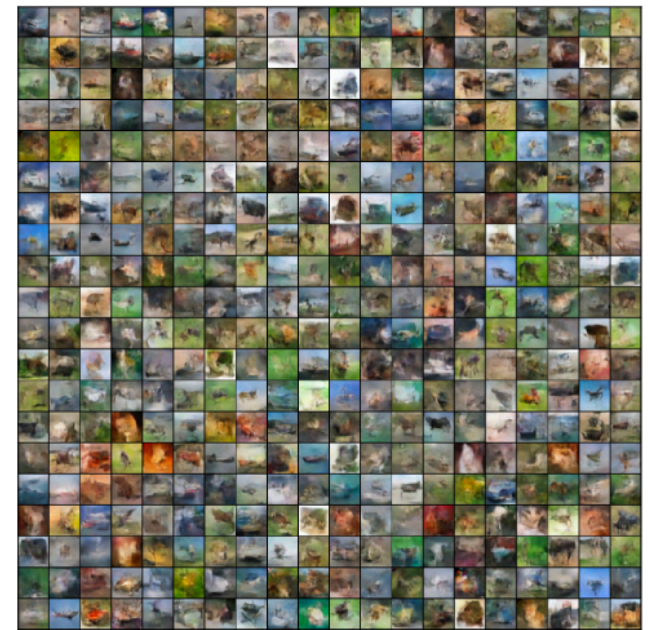
- L_2 distance leads to **blur images**.



(a) MNIST



(b) TFD



(c) CIFAR-10

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Generative Latent Nearest Neighbors (GLANN)

Main Drawbacks of GLO and IMLE

- GLO
 - The distribution of noise space Z is unknown.
 - There is no principled way to sample new images.
 - Fitting Gaussian to the noise distribution in the paper does not synthesis high quality images.
- IMLE
 - IMLE is sensitive to the distance metric used.
 - Using L_2 distance on image pixel space causes blurry synthesised images.
 - Computing nearest neighbour for each sample in the image space is costly.

Generative Latent Nearest Neighbors (GLANN)

Approach

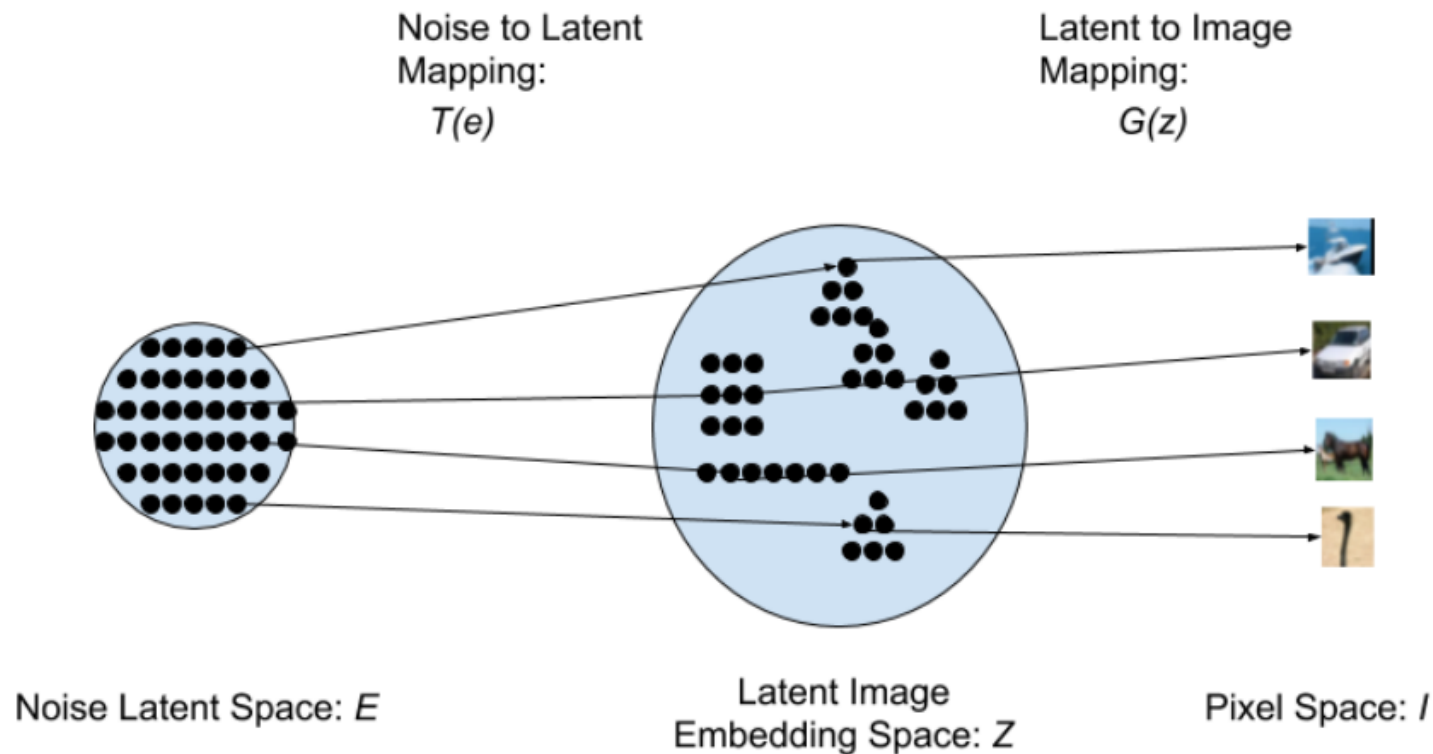
- Linear arithmetic in GLO's noise space can semantically manipulate the image.
- That means Euclidean metric in GLO's noise space is semantically meaningful.
- Why not use GLO to learn a good latent distribution, then apply IMLE to sample from noise?

Generative Latent Nearest Neighbors (GLANN)

Approach

Stage 2: IMLE learns e to z

Stage 1: GLO learns z to X



Generative Latent Nearest Neighbors (GLANN)

Approach

- Stage 1: Latent2Image using GLO
 - $z \mapsto x$
 - Use VGG perceptual loss function

$$\arg \min_{\tilde{G}, \{z_i\}} \sum_i \ell_{\text{perceptual}}(\tilde{G}(z_i), x_i) \quad \text{s.t.} \quad \|z_i\| = 1$$

- Stage 2: Noise2Latent using IMLE
 - $e \mapsto z$
 - Using Euclidean distance in the latent space
- Sampling new images
 - $e \sim N(0, I)$
 - $e \mapsto z \mapsto x$

Generative Latent Nearest Neighbors (GLANN)

Approach

- Stage 1: Latent2Image using GLO
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$$\tilde{z}_m = T(e_m)$$

- Stage 2: Noise2Latent using IMLE
 - $e \mapsto z$
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$$e_t = \arg \min_{e_m} \|z_t - T(e_m)\|_2^2$$

$$T = \arg \min_{\tilde{T}} \sum_t \|z_t - \tilde{T}(e_t)\|_2^2$$

- Sampling new images
 - $e \sim N(0, I)$
 - $e \mapsto z \mapsto x$

Generative Latent Nearest Neighbors (GLANN)

Approach

- Stage 1: Latent2Image using GLO
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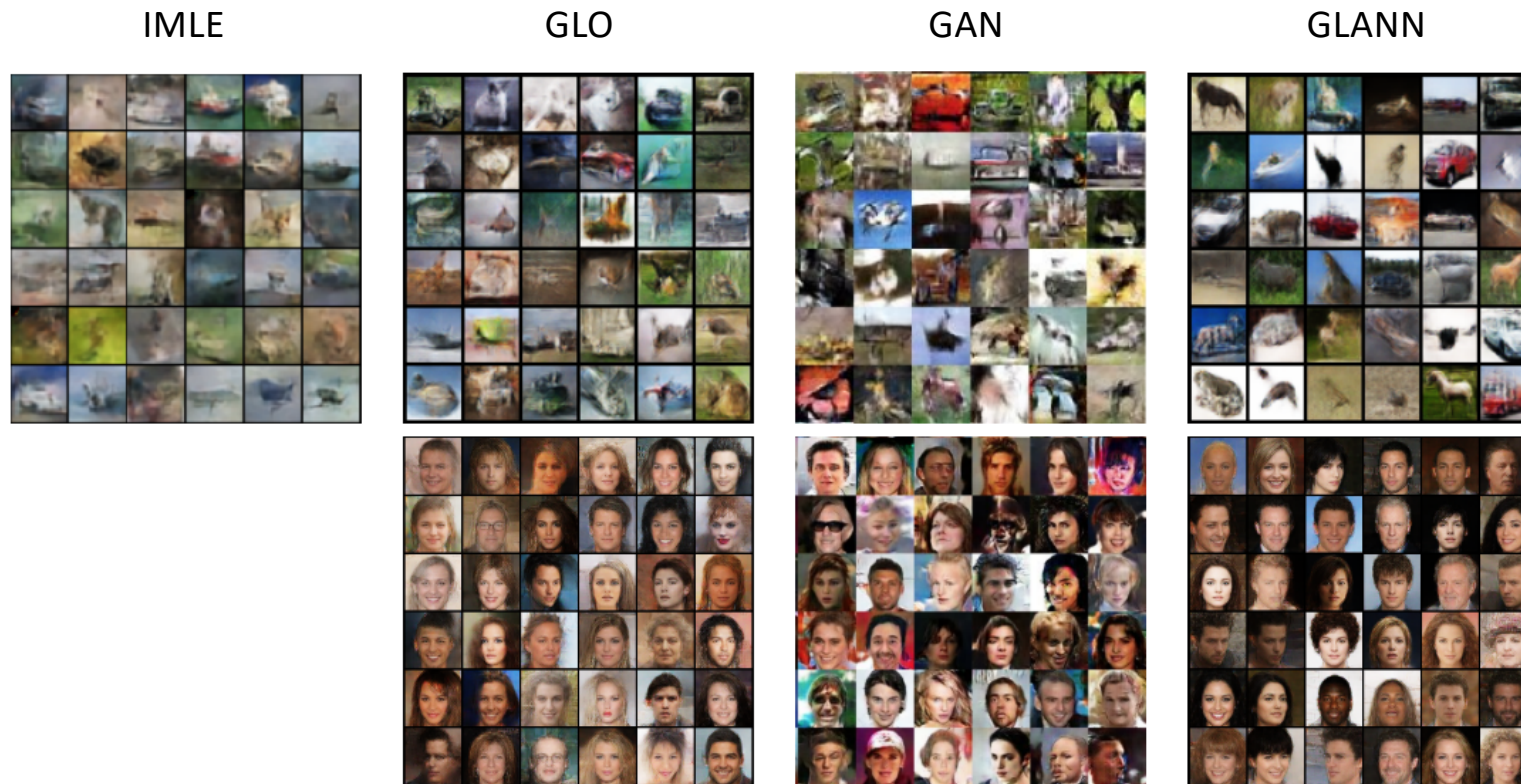
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- Sampling new images
 - $e \sim N(0, I)$
 - $e \mapsto z \mapsto x$

Generative Latent Nearest Neighbors (GLANN)

Results



Generative Latent Nearest Neighbors (GLANN)

Results



Interpolation on CelebA-HQ 1024*1024



Generating 3D data

- Generative Latent Optimisation, GLO
- Implicit Maximum Likelihood Estimation, IMLE
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- **Discussion**

Discussion: What is the Ideal Generative Model?

- Explicit inverse/encode $E: x \rightarrow z, G: z \rightarrow x$
- Interpolation in latent space (prior distribution)
- Avoid mode collapse
- Fast training
- Generate high-dimensional, high-quality data
- Disentanglement
- ...

Summary



- Generative [Latent Optimisation](#), GLO 2018
- [Implicit](#) Maximum Likelihood Estimation, IMLE 2018
- Generative [Latent Nearest Neighbors](#), GLANN 2019
- Discussion

Thanks