Autoregressive Models

- Pixel RNN/CNN, WaveNet

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Autoregressive Models

- Assumption of Autoregressive Models
- Fully visible Sigmoid Belief Network (FVSBN)
- Neural Autoregressive Density Estimation (NADE)
- Real-valued Neural Autoregressive Density Estimation (RNADE)
- Autoregressive Autoencoders
- Masked Autoencoder for Distribution Estimation (MADE)
- Recurrent Neural Networks
- Pixel RNN: Pixel CNN, Row LSTM and Diagonal BiLSTM
- Gated PixelCNN
- WaveNet

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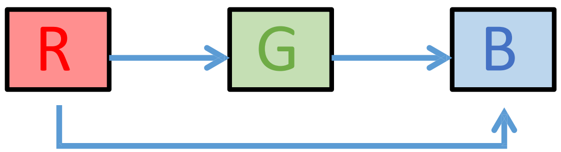
Autoregressive Image Modeling

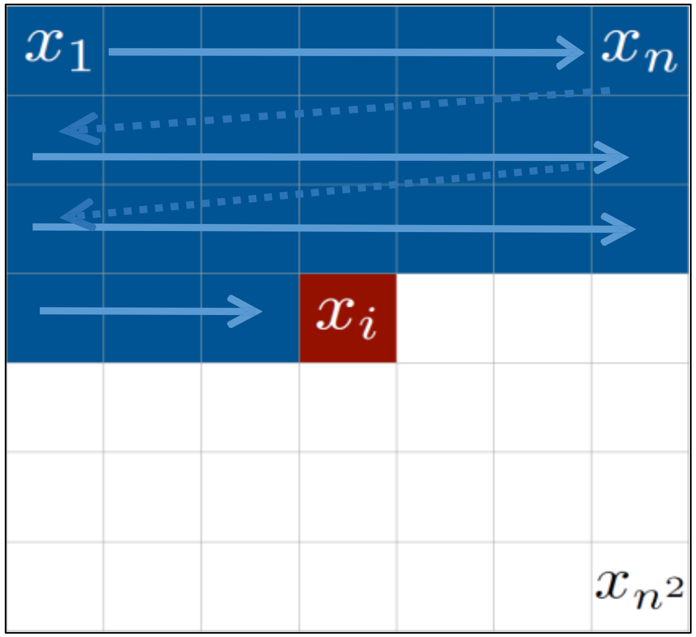
-Autoregressive models train a network that model the conditional distribution of every individual pixel given previous pixels (raster scan order dependencies).

$$p(\mathbf{x}) = \prod_{i=1}^{n^2} p(x_i | x_1, ..., x_{i-1}).$$



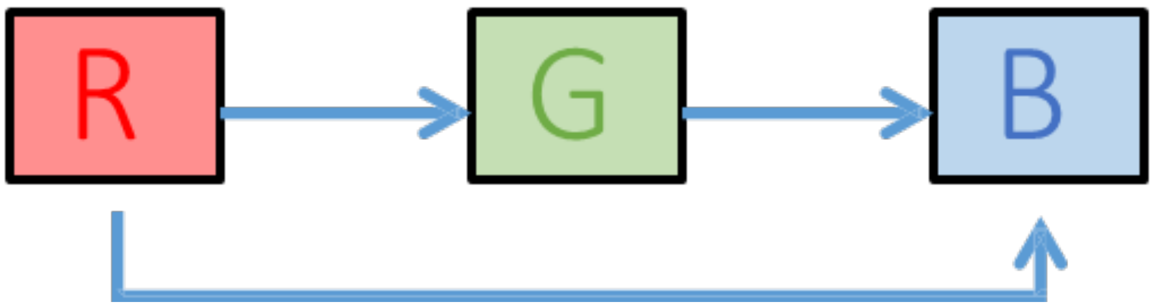
Autoregressive Image Modeling

Sequentially predict pixels rather than predicting the whole image at once (like as GAN, VAE)



Autoregressive Image Modeling

- For color image, 3 channels are generated successive conditioning, blue given red and green, green given red, and red given only the pixels above and to the left of all channels



Pixel Recurrent Neural Networks

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Google DeepMind

PixelCNN
PixelRNN
- Row LSTM
- Diagonal BiLSTM

（ICML 2016 Best Paper）

Conditional Image Generation with PixelCNN Decoders

Gated PixelCNN

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WaveNet

WaveNet: A Generative Model for Raw Audio

Aäron van den Oord **Sander Dieleman** **Heiga Zen**†
Karen Simonyan **Oriol Vinyals** **Alex Graves**
Nal Kalchbrenner **Andrew Senior** **Koray Kavukcuoglu**

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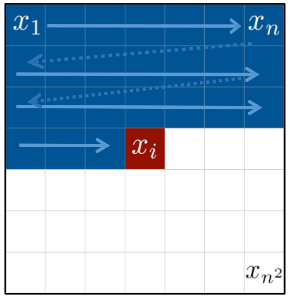
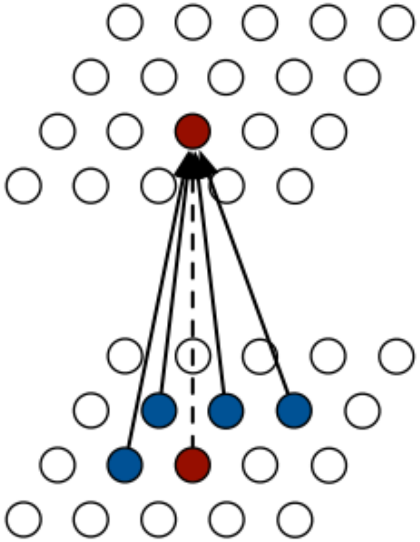
PixelCNN Network Structure



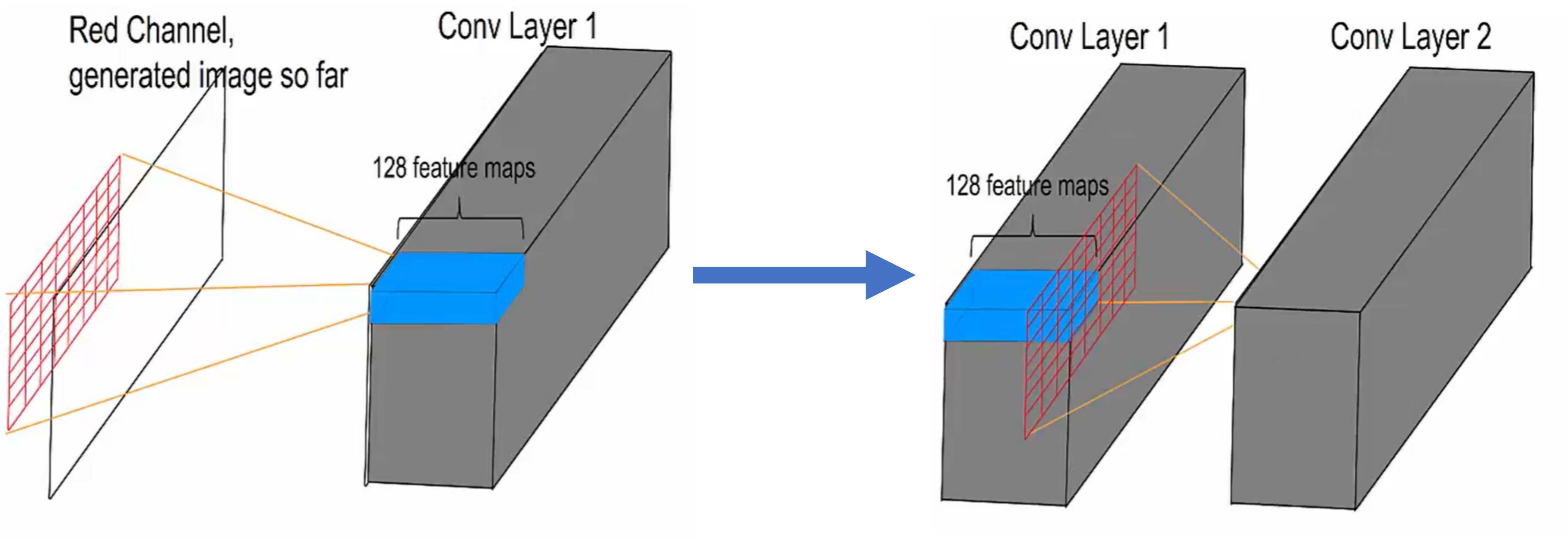


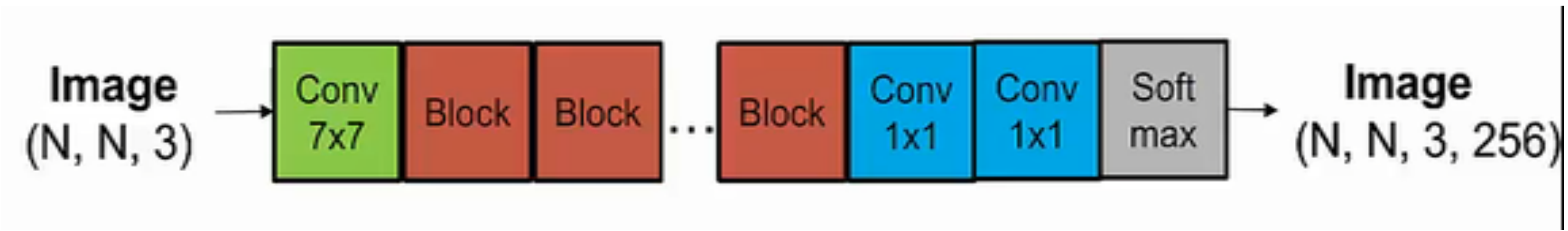
PixelCNN

Goal:

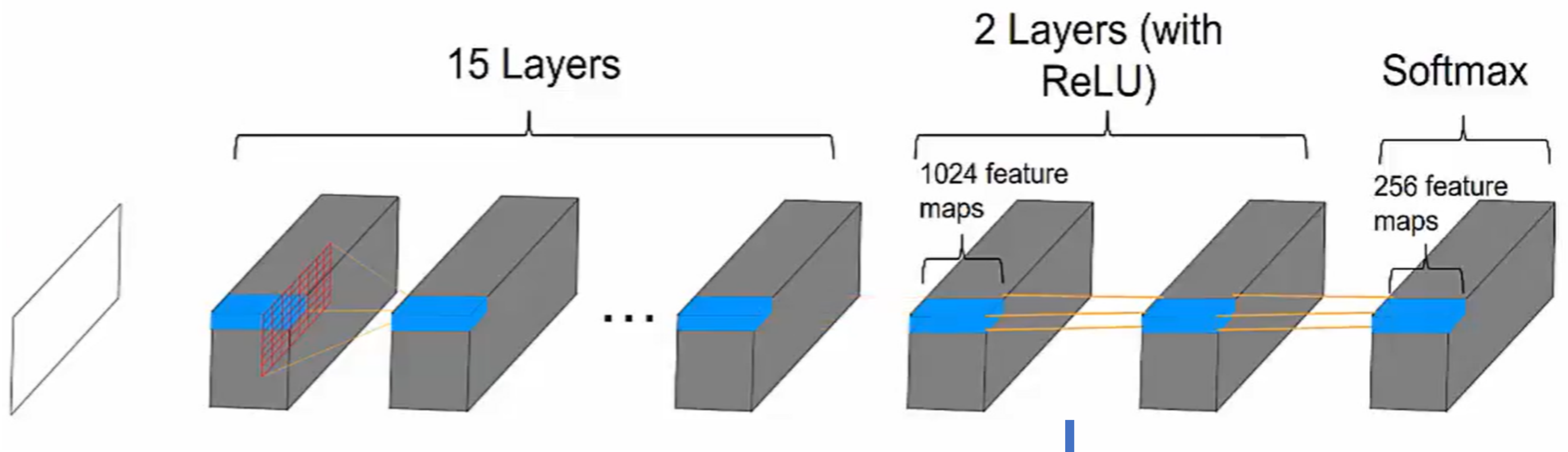
- Use the neighbor pixels to predict the new pixel

PixelCNN Network Structure



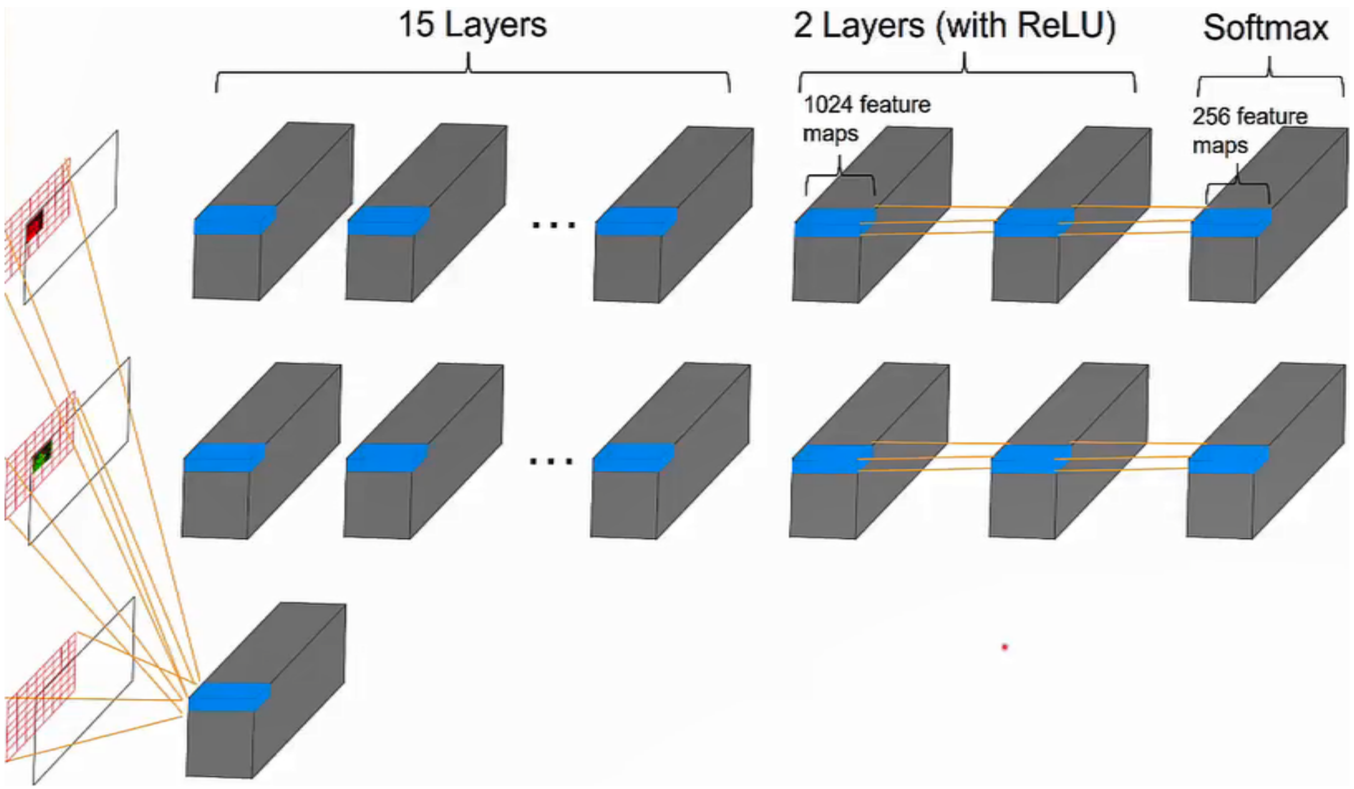


PixelCNN Network Structure

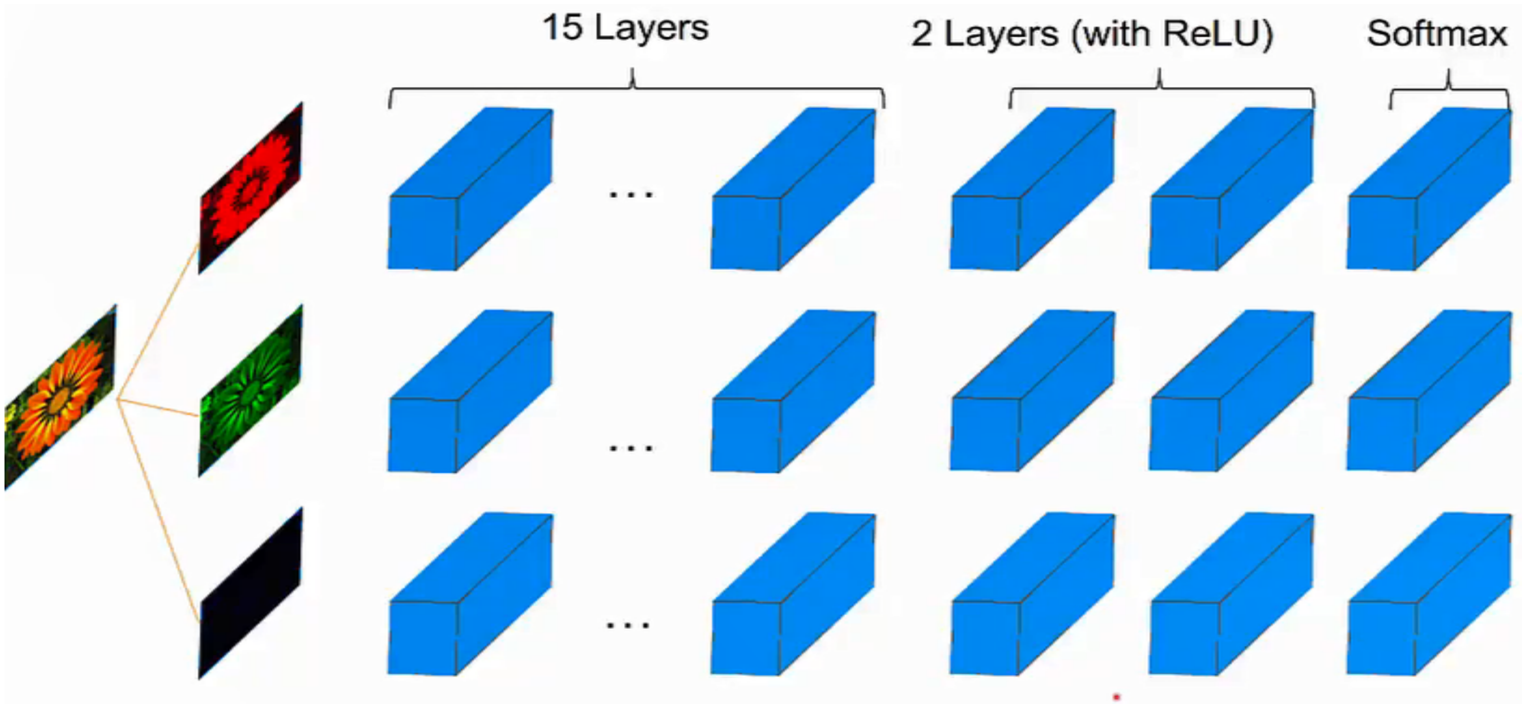


Two one-by-one Convs

PixelCNN Network Structure: Repeat for 3 times for RGB



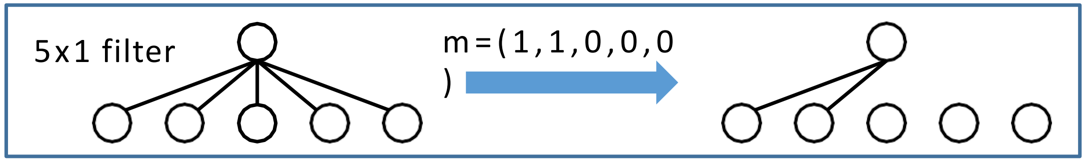
When training, something differs…



Details of “Masked Convolution” & “Blind Spot”

- To generate next pixel, the model can only condition on the previously generated pixels.
- Then, to make sure CNN can only use information about pixels above and to the left of current pixel, the filters of the convolution need to be masked.

Case 1D

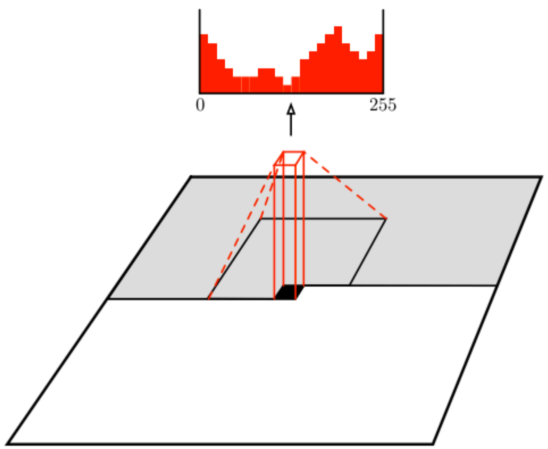


- Right figure shows 5x1 convolutional filters after multiplying them by mask.
- The filters connecting the input layer to the first
- hidden layer are in this case multiplied by m=(1,1,0,0,0), to ensure the model is causal.

Details of “Masked Convolution” & “Blind Spot”

Case 2D

- In case of 2D, PixelCNNs have a **blind spot** in the receptive field that cannot be used to make predictions.
- Rightmost figure shows the growth of the masked receptive field. (3 layered network with 3x3 conv filters)



5x5 filter

1	1	1	1	1
1	1	1	1	1
1	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Details of "Masked Convolution" & "Blind Spot"

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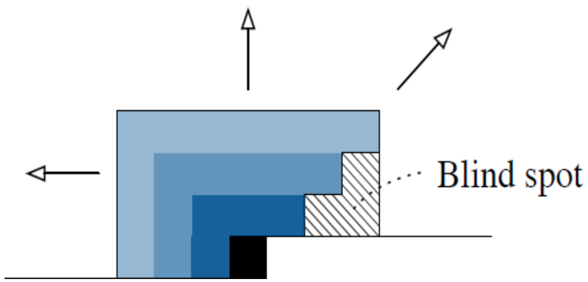
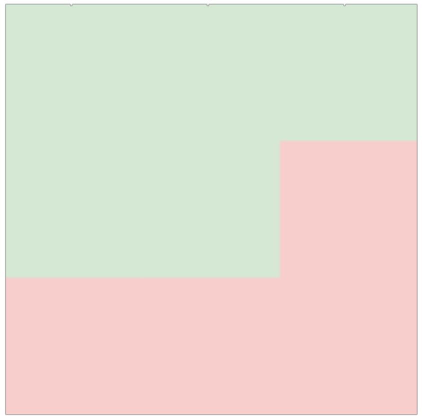
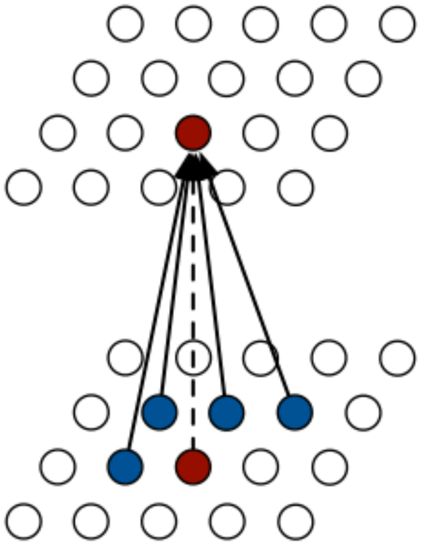


3x3 filter, 3 layered



Pixel CNN Drawbacks



PixelCNN

Drawbacks:

- Sequential generation is SLOW
- Blind spot problem

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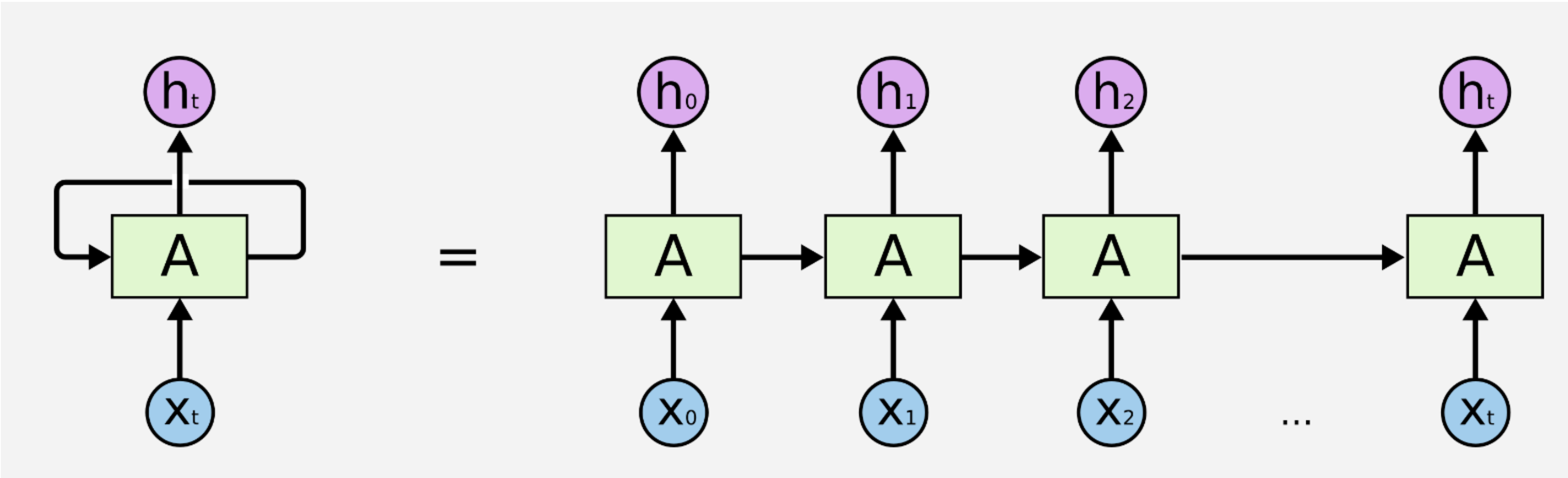
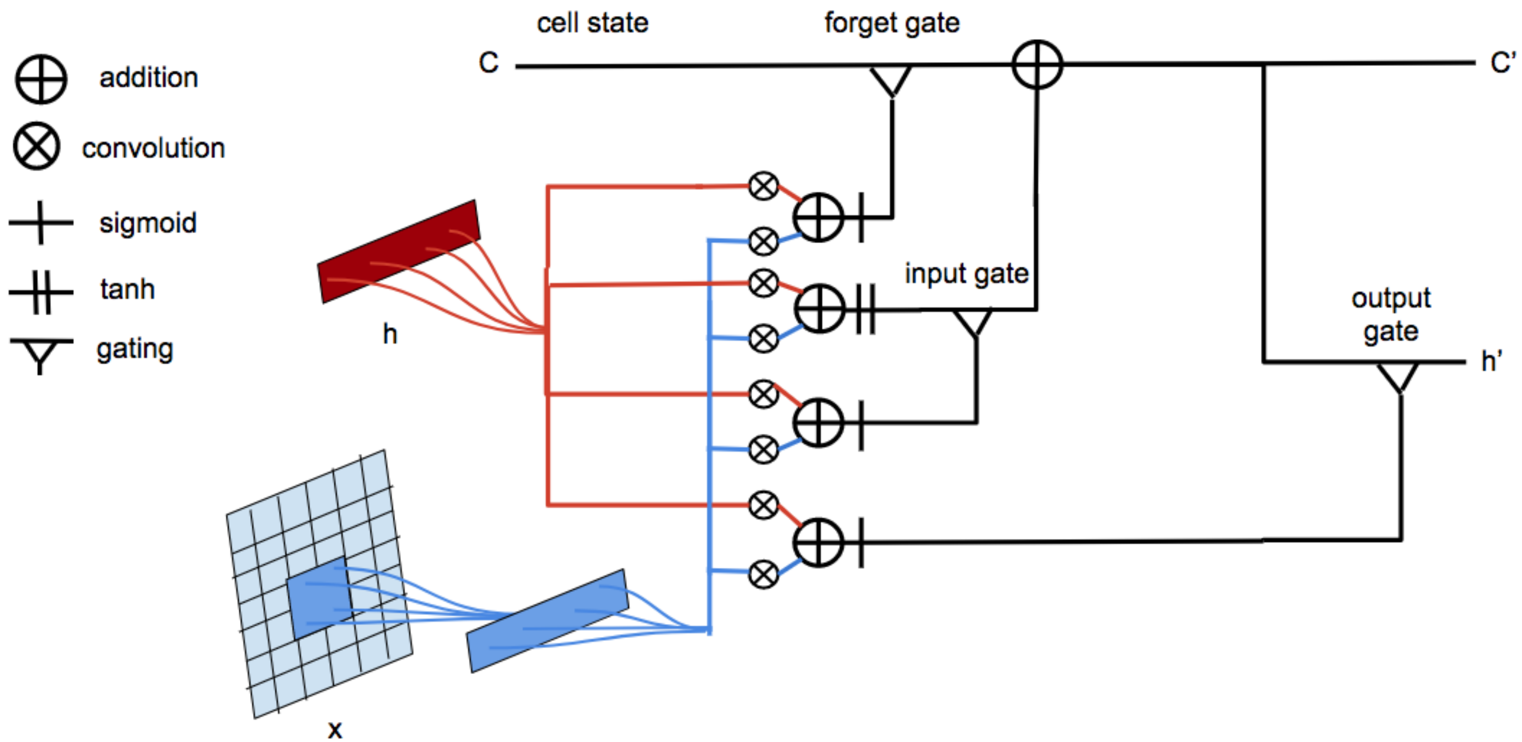
Recap RNN…



Convolutional LSTM…



With the Convolutional LSTM, we change LSTM Equations from … into …

LSTM Equations

$$i = \sigma(x_i U^i + \mathbf{h}_{i-1} W^i)$$
$$f = \sigma(x_i U^f + \mathbf{h}_{i-1} W^f)$$
$$o = \sigma(x_i U^o + \mathbf{h}_{i-1} W^o)$$
$$g = \ tanh(x_i U^g + \mathbf{h}_{i-1} W^g)$$

Gates - Control how much information is allowed through

$$c_i = c_{i-1} \circ f + g \circ i$$
$$\mathbf{h}_i = \tanh(c_i) \circ o$$

States - Hold information about all time steps up till now {0, i, …, i-1, i}

LSTM Equations

Like Convolutional LSTM - replaced fully-connected layer with convolutional layer

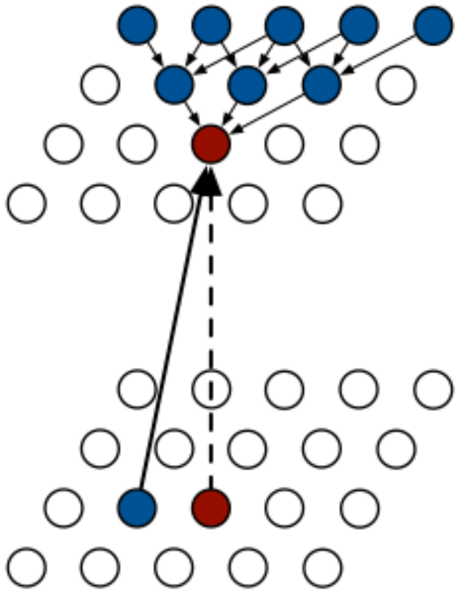
$$i = \sigma(x_i \cancel{U^i} + \mathbf{h}_{i-1} \cancel{W^i})$$
$$f = \sigma(x_i \cancel{U^f} + \mathbf{h}_{i-1} \cancel{W^f})$$
$$o = \sigma(x_i \cancel{U^o} + \mathbf{h}_{i-1} \cancel{W^o})$$
$$g = \ tanh(x_i \cancel{U^g} + \mathbf{h}_{i-1} \cancel{W^g})$$

$$[\mathbf{o}_i, \mathbf{f}_i, \mathbf{i}_i, \mathbf{g}_i] = \sigma(\underline{\mathbf{K}^{ss} \circledast} \mathbf{h}_{i-1} + \underline{\mathbf{K}^{is} \circledast} \mathbf{x}_i)$$

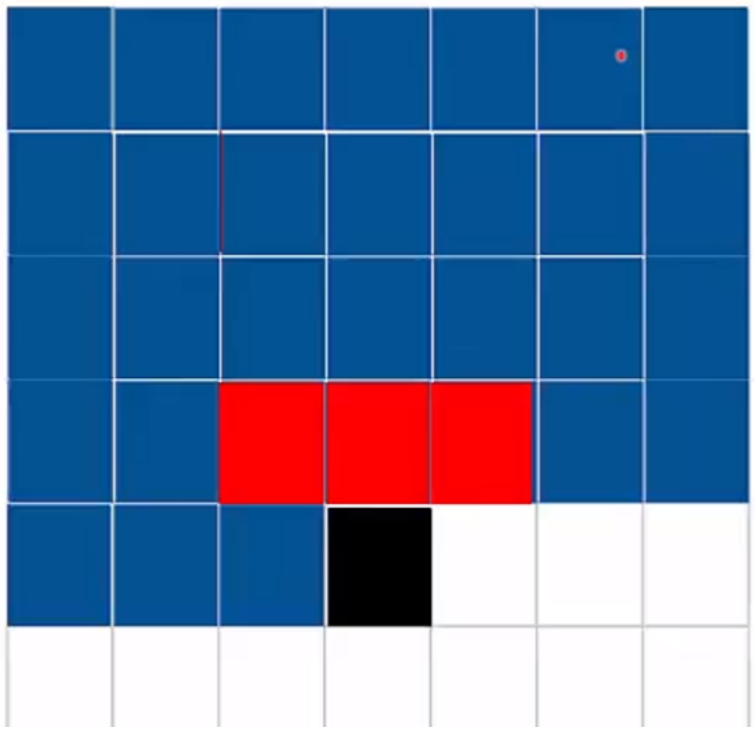
$$c_i = c_{i-1} \circ f + g \circ i$$
$$\mathbf{h}_i = \tanh(c_i) \circ o$$

$$\mathbf{c}_i = \mathbf{f}_i \odot \mathbf{c}_{i-1} + \mathbf{i}_i \odot \mathbf{g}_i$$
$$\mathbf{h}_i = \mathbf{o}_i \odot \tanh(\mathbf{c}_i)$$

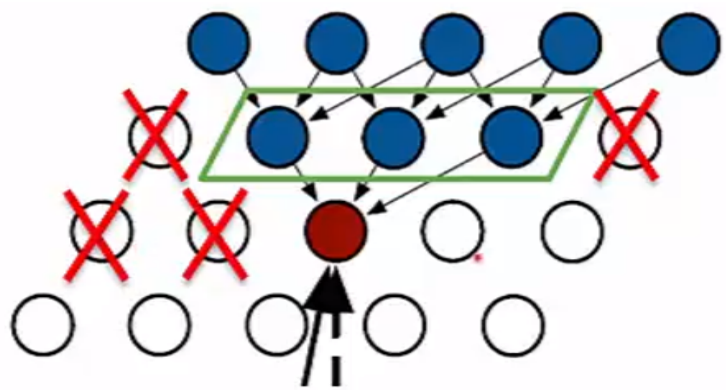
PixelRNN 1: Row LSTM



Row LSTM



So the receptive field…



Triangular receptive field

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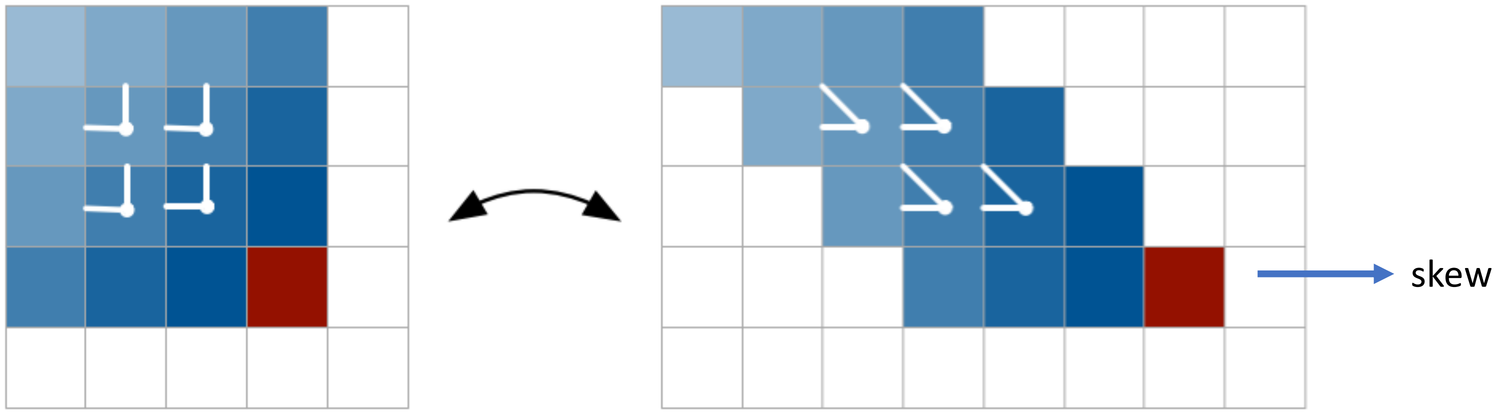
PixelRNN2: Diagonal BiLSTM



In the Diagonal BiLSTM, to allow for parallelization along the diagonals, the input map is skewed by offseting each row by one position with respect to the previous row. When the spatial layer is computed left to right and column by column, the output map is shifted back into the original size. The convolution uses a kernel of size 2×1.

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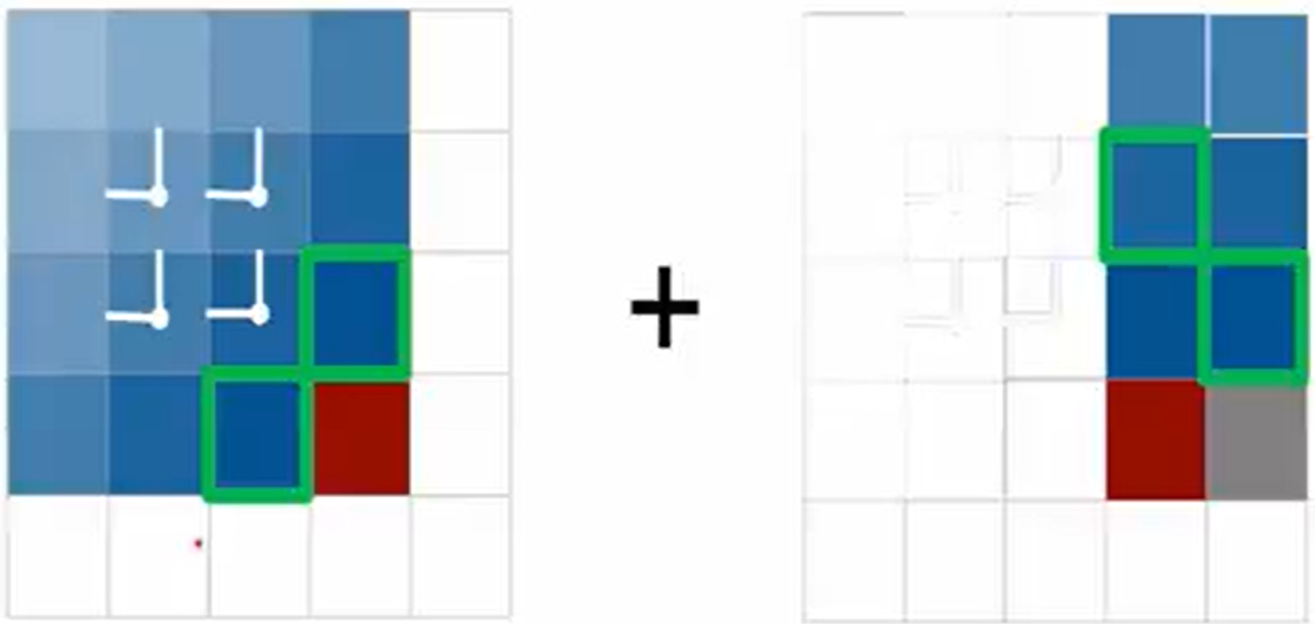
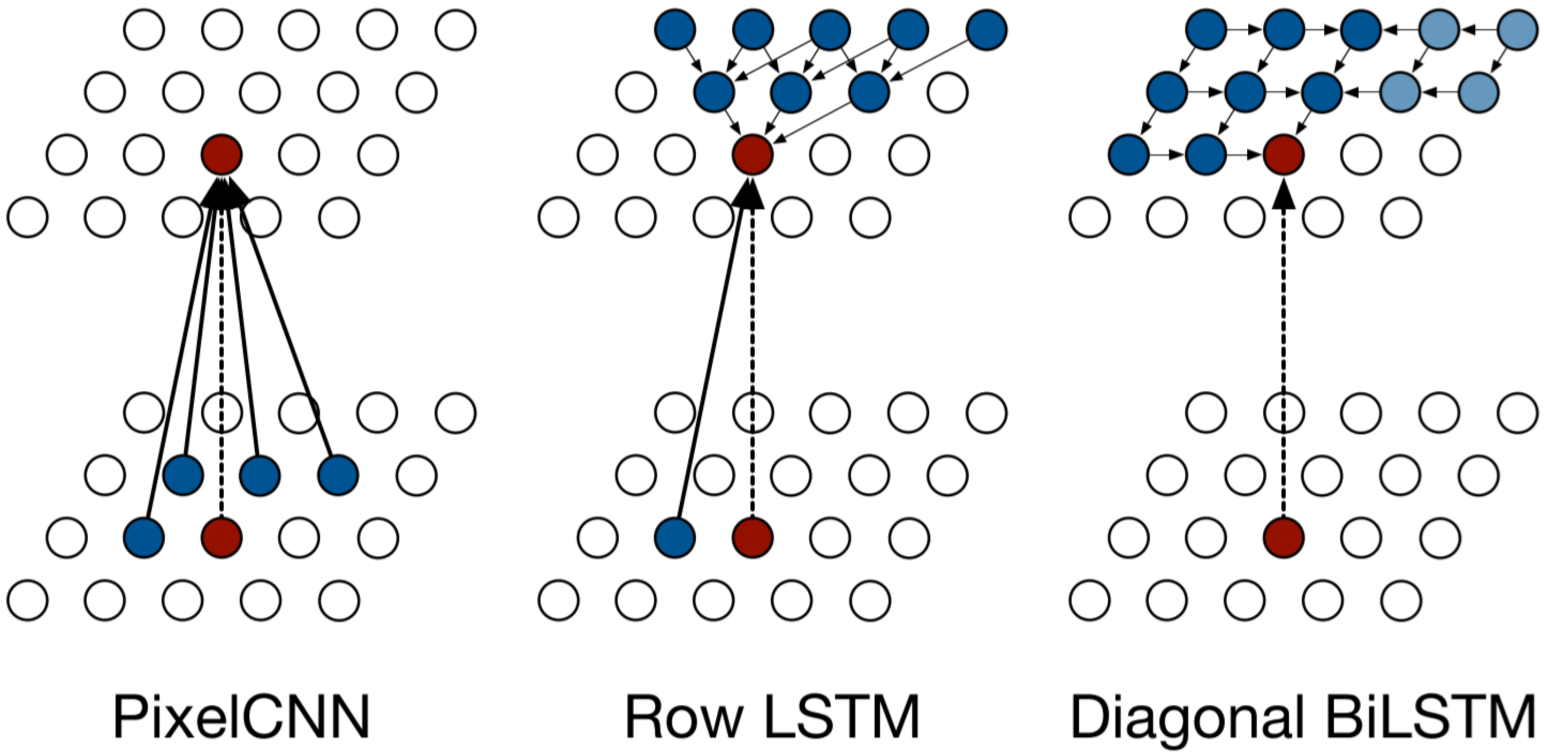
Diagonal BiLSTM: why “Bi”?

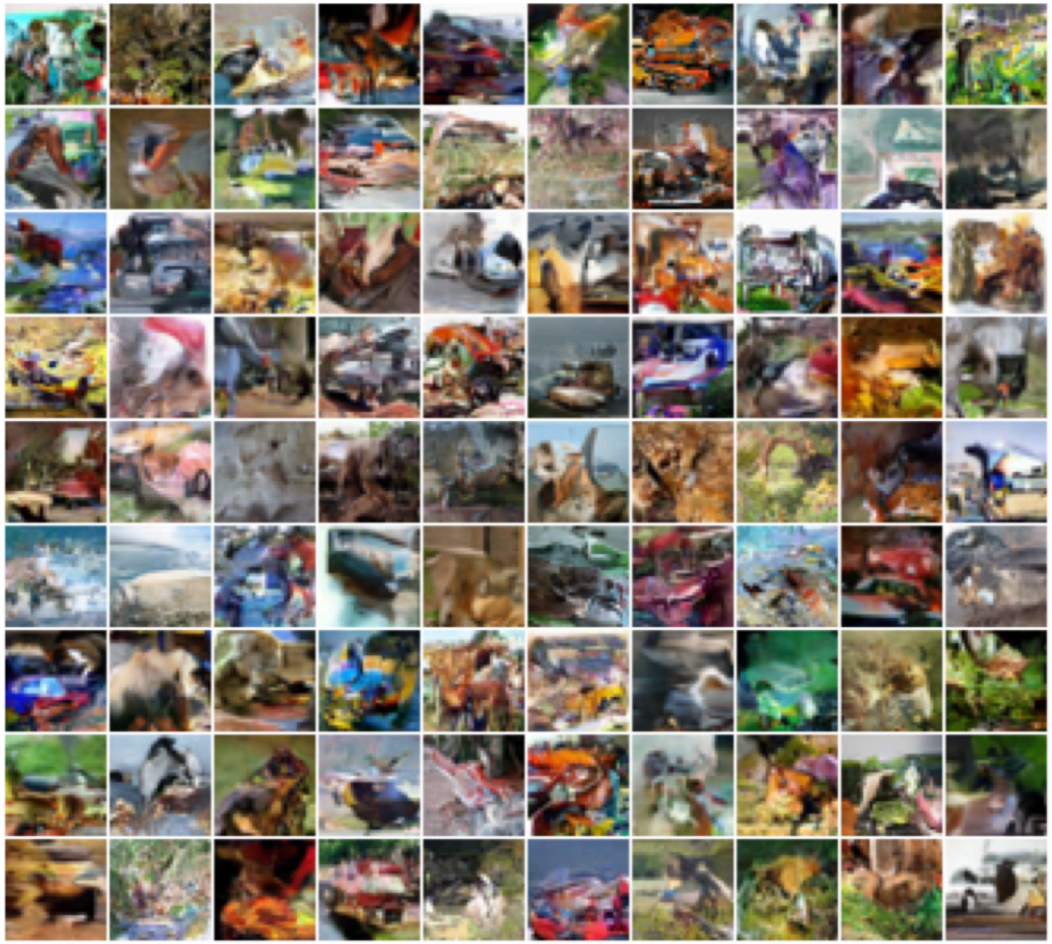


Comparison between the 3 network above…

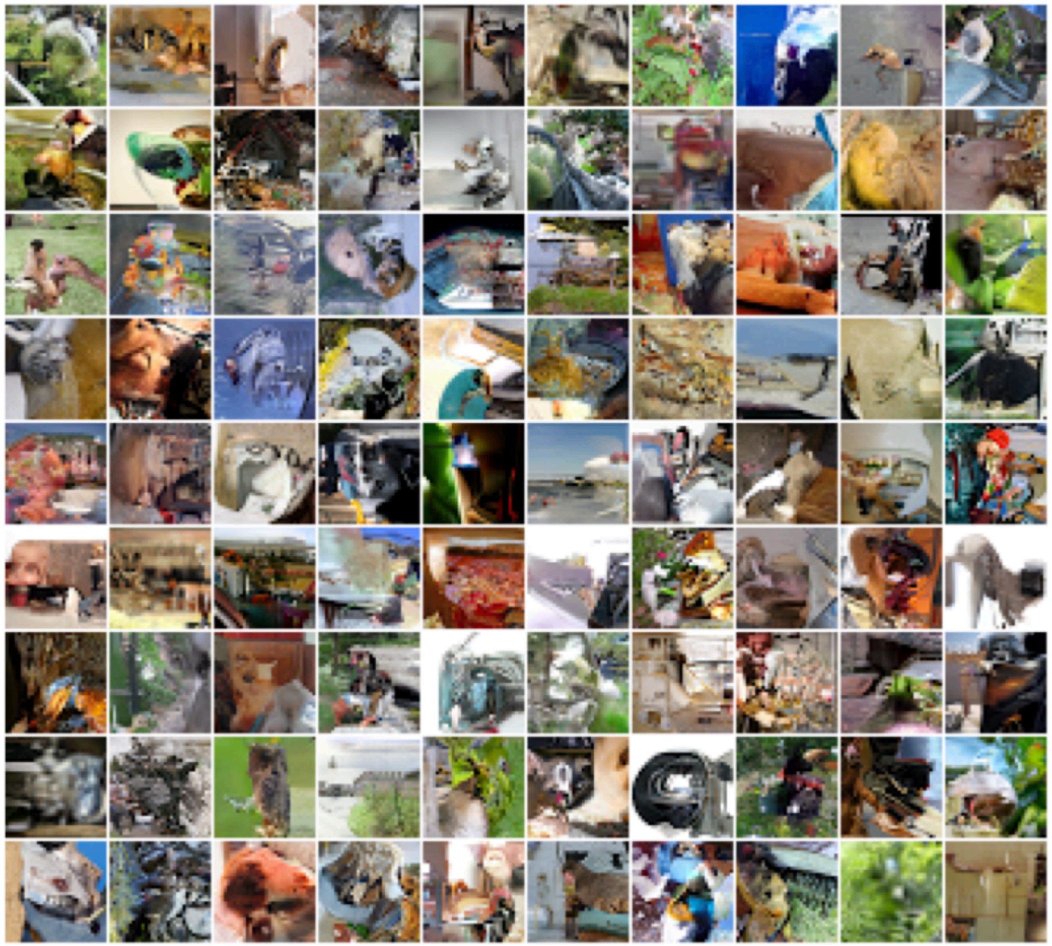


Results of PixelRNN

CIFAR-10



ImageNet 32x32



Results of PixelRNN

Figure 1. Image completions sampled from a PixelRNN.

PixelRNN vs. PixelCNN

	PixelRNN	PixelCNN
Pros.	• Effectively handles long-range dependencies • **Good performance**	Convolutions are easier to parallelize $\Rightarrow$ **Much faster** to train
Cons.	• Each state needs to be computed sequentially. • **Computationally expensive**	Bounded receptive field $\Rightarrow$ **Inferior performance** **Blind spot problem** (due to the masked convolution) needs to be eliminated.

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Gated PixelCNN

An improved version of PixelCNN, major improvements are as follows：

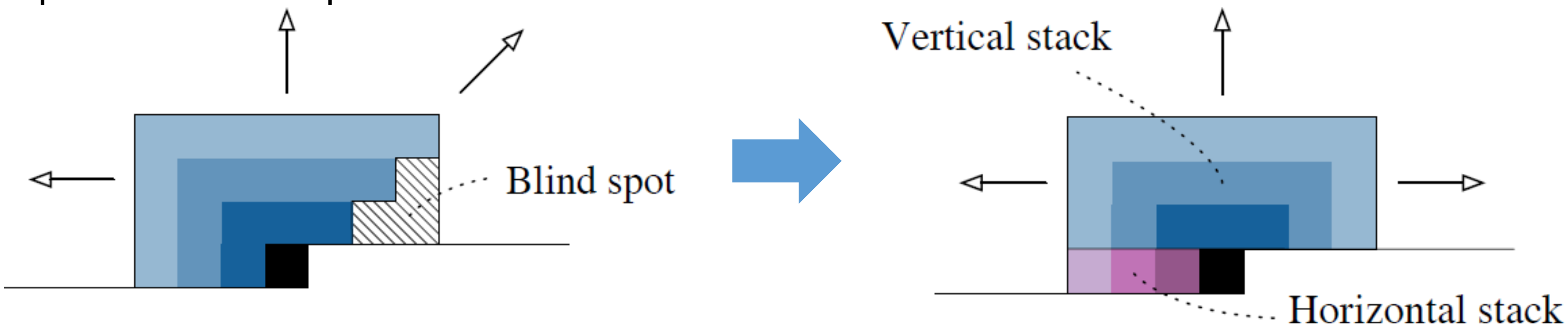
- Removal of blind spots in the receptive field by combining **the horizontal stack** and **the vertical stack**.
- Replacement of the ReLU activations between the masked convolutions in the original PixelCNN with the **gated activation unit**.

Gated PixelCNN

- Given a latent vector, they modeled the conditional distribution of images, **Conditional PixelCNN**.
    - conditioning on class-label
    - conditioning on embedding from trained model
- From a convolutional auto-encoder, they replaced the deconvolutional decoder with conditional PixelCNN, named **PixelCNN Auto-Encoders**

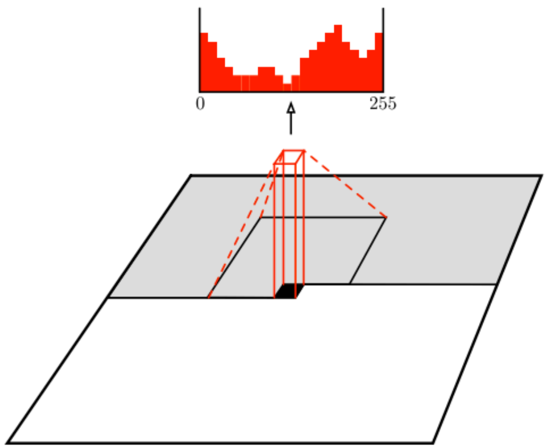
First improvement:
Horizontal stack and Vertical stack

- The removal of blind spots in the receptive field are important for PixelCNN's performance, because the blind spot can cover as much as a quarter of the potential receptive field.



- The **vertical stack** conditions on all rows above the current row.
- The **horizontal stack** conditions on current row.

First improvement:
Horizontal stack and Vertical stack



Mask for PixelCNN

1	1	1	1	1
1	1	1	1	1
1	1	0	0	0
0	0	0	0	0
0	0	0	0	0

They cause blind spot

Mask for Gated PixelCNN

Vertical stack

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	0	0	0	0
0	0	0	0	0

Why they can be 1?

Horizontal stack

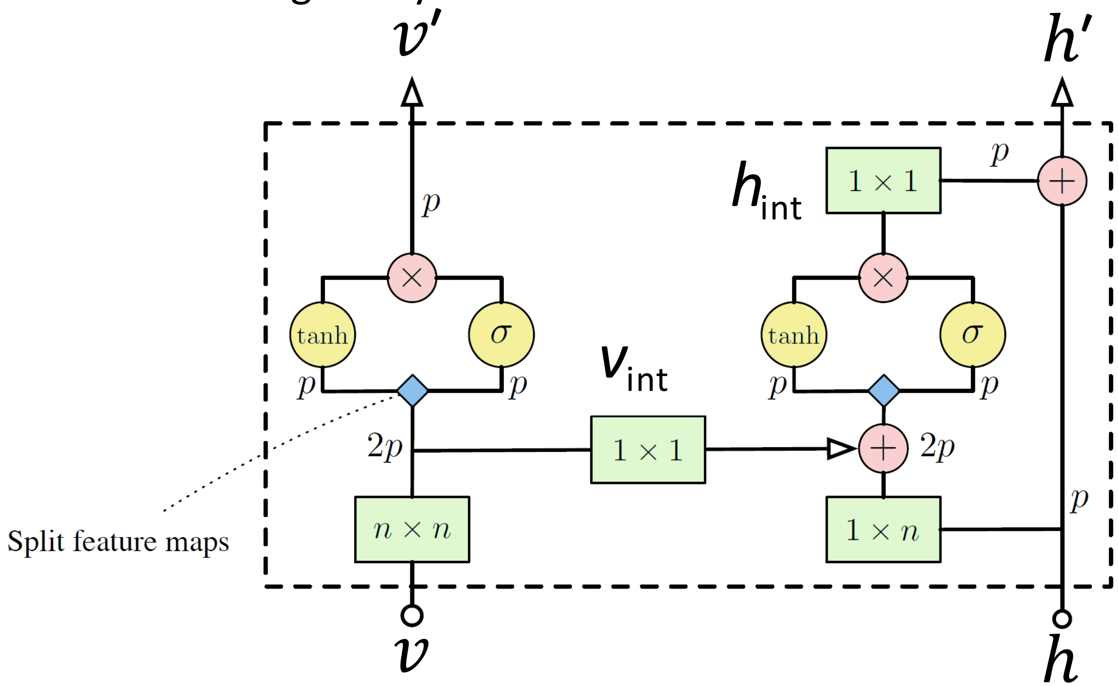
1	1	0	0	0

Second improvement:
Gated Activation and Architecture

- Gated activation unit: $\mathbf{y} = \tanh(W_{k,f} * \mathbf{x}) \odot \sigma(W_{k,g} * \mathbf{x})$

(σ: sigmoid, k: number of layer, ⊙: element-wise product, * : convolutional operator)

- Single layer block of a GatedPixelCNN



- Masked convolutions are shown in green.
- Element-wise operations are shown in red.
- Convolutions with W_f, W_g are combined into a single operation shown in blue.

p = #feature maps
v = vertical activation maps
h = horizontal activation maps

Details of Gated PixelCNN architecture

- Break down operations into four steps.

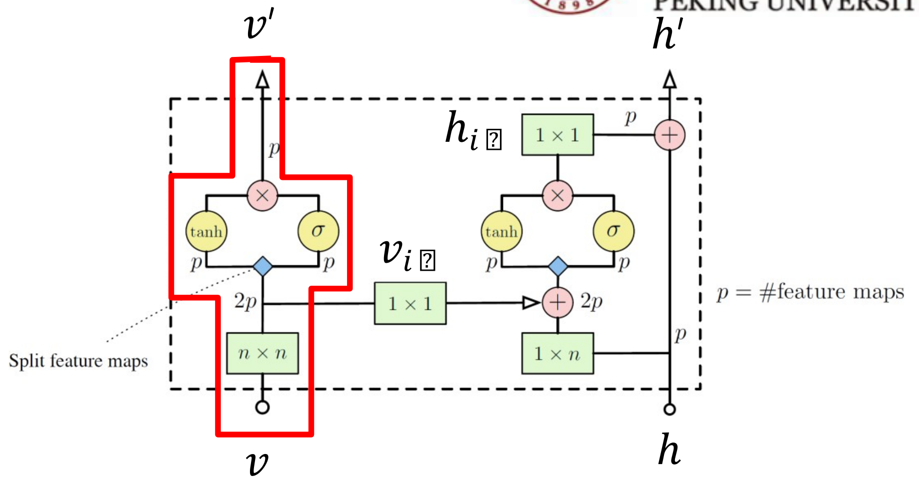
① Calculate vertical feature maps

… n×n convolutions are calculated with gated activation.

Input: v (= input image if 1st layer)

Output: v'

(ex. n = 3)



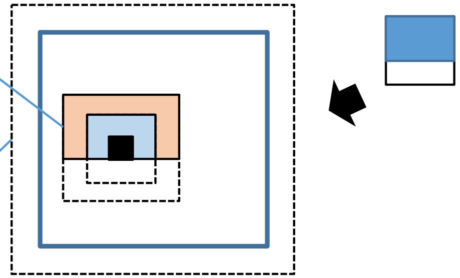
Two types of equivalent implementation :

Feature map

3x3 masked filters

receptive field

(1,1,1,1) zero padding

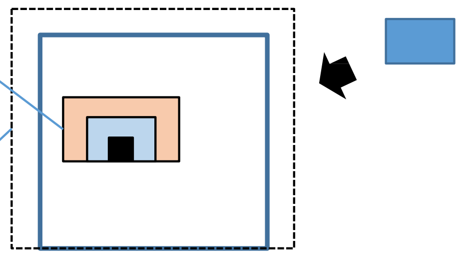


Feature map

2x3 filters

receptive field

(1,0,1,1) zero padding



Next problem:

In this case, (i, j)th pixel depends on (i, j+k)th (future) pixels

Solution:

Shift down vertical feature maps when to feed into horizontal stack.

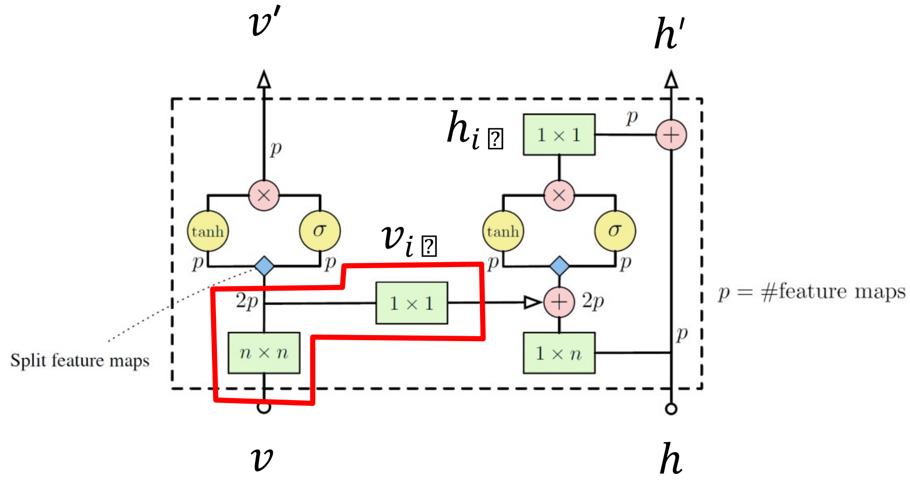
Details of Gated PixelCNN architecture

② Feed vertical maps into horizontal stack

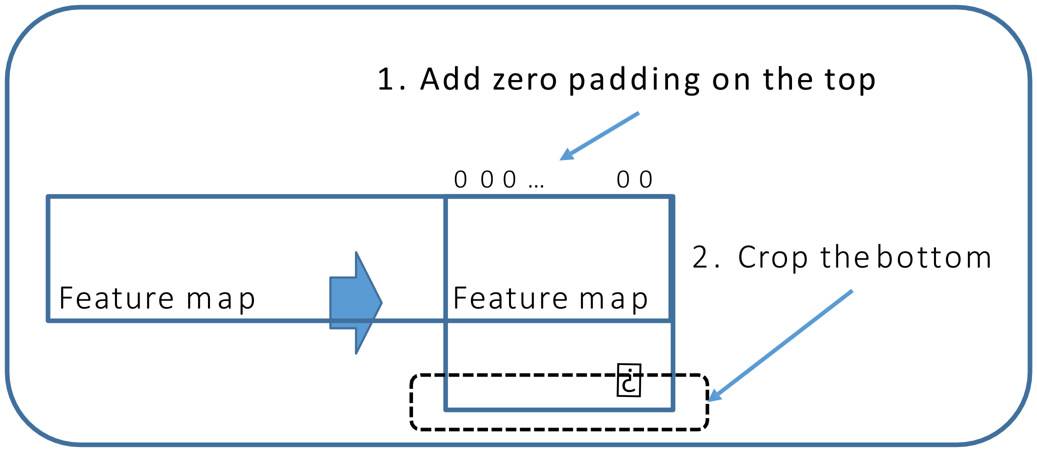
1. n x n masked convolution
2. shifting down operation (as below)
3. 1 x 1 convolution

Input: v (= input image if 1st layer)

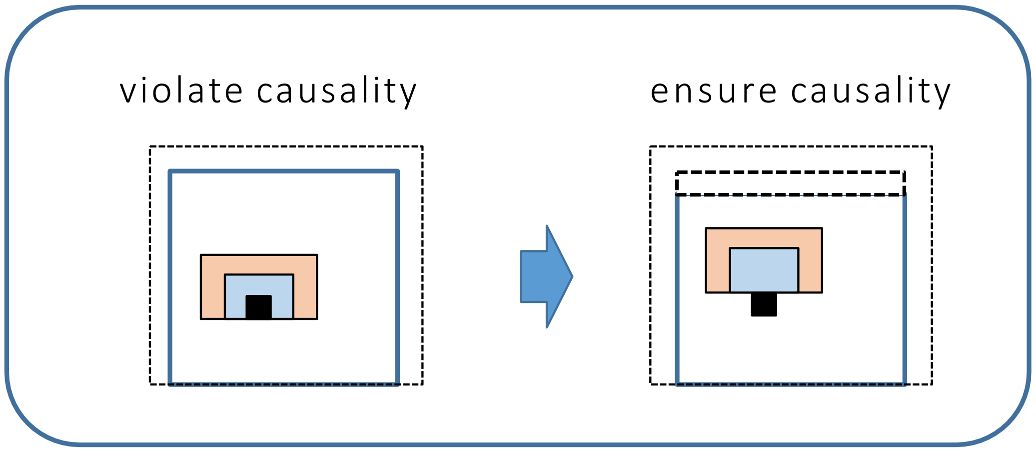
Output: v_{int}



Shift down vertical feature maps when to feed into horizontal stack.



Left operations can be interpreted as below.



Details of Gated PixelCNN architecture

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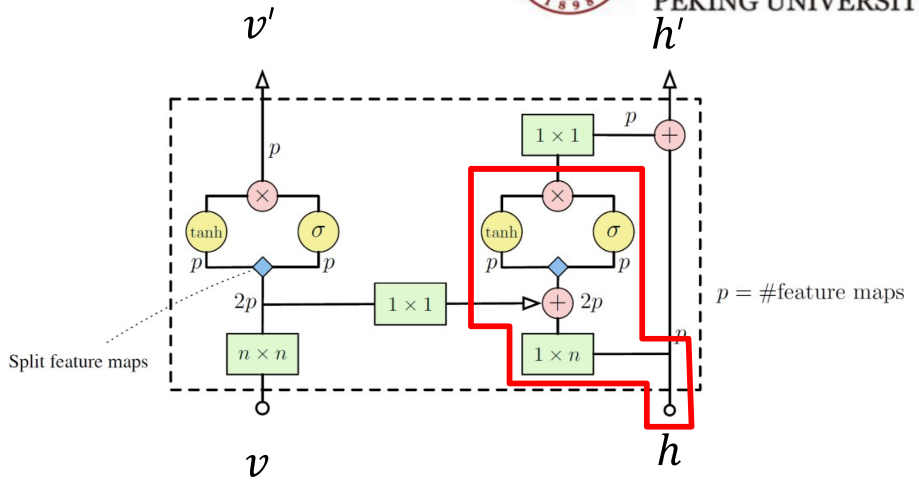
③ Calculate horizontal feature maps

… 1×n convolutions are calculated with gated activation.
(vertical maps are added before activation.)

Input: v_{int} , h (input image if 1st layer)

Output: h_{int}

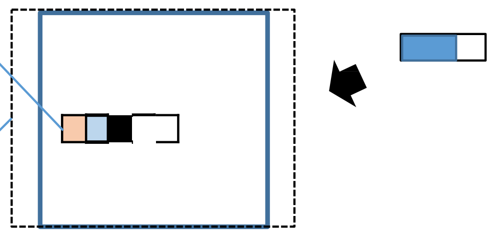
(ex. n = 3)



Feature map 1x3 masked filters

receptive field

(0,0,1,1)
zero padding

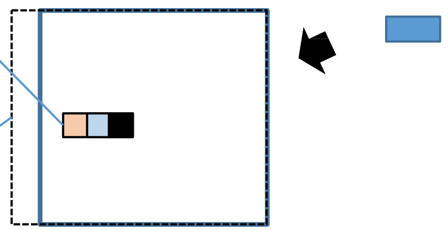


Two types of equivalent implementation :

Feature map 1x2 filters

receptive field

(0,0,1,0)
zero padding



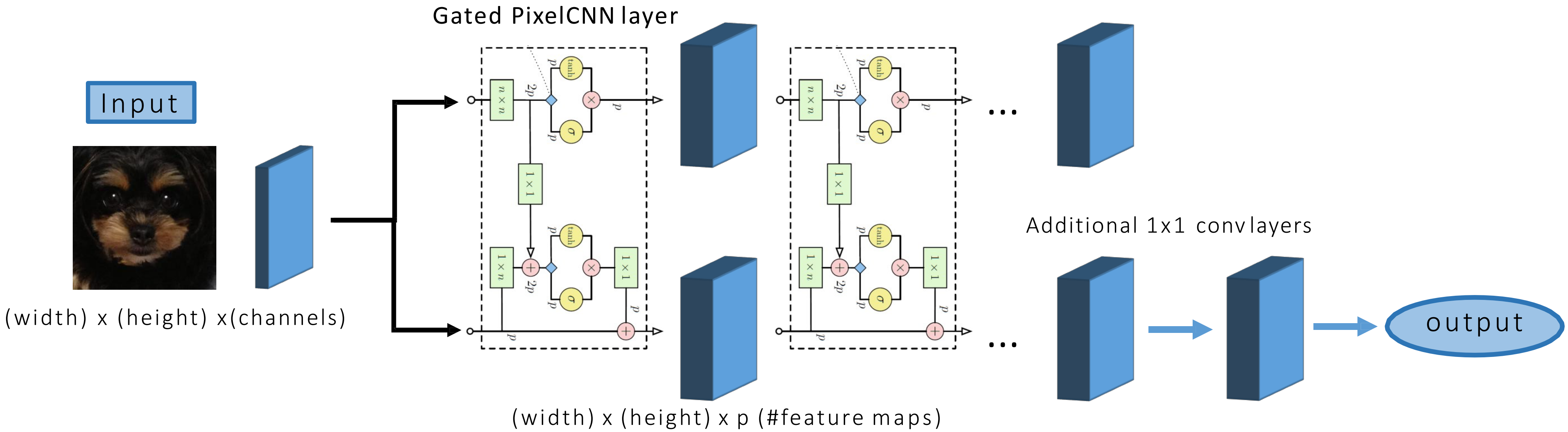
Next problem:
➢ Mask A vs Mask B

- Mask A (restrict connection from itself) is applied to only to the first convolution.
- Mask B (allow connection from itself) is applied to all the subsequent convolution.

Output layer and whole architecture

- Output layer
  - Using a softmax on discrete pixel values ([0-255] = 256 way) instead of a mixture density approach. (same approach as PixelRNN)
  - Although without prior information about the meaning or relations of the 256 color categories, the distributions predicted by the model are meaningful.

- Whole architecture



Third improvements:
Conditional PixelCNN & PixelCNN AutoEncoder

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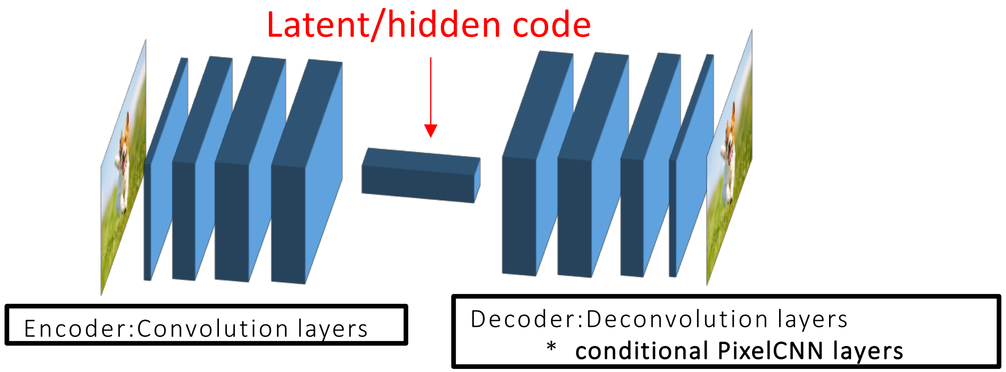
■ Coniditional PixelCNN

	original	conditional
Model	$p(\mathbf{x}) = \prod_{i=1}^{n^2} p(x_i \mid x_1, ..., x_{i-1}).$	$p(\mathbf{x} \mid \mathbf{h}) = \prod_{i=1}^{n^2} p(x_i \mid x_1, ..., x_{i-1}, \underline{\mathbf{h}})$
Gated activation unit	$\mathbf{y} = \tanh(W_{k,f} * \mathbf{x}) \odot \sigma(W_{k,g} * \mathbf{x})$	$\mathbf{y} = \tanh(W_{k,f} * \mathbf{x} + \underline{V_{k,f}^T \mathbf{h}}) \odot \sigma(W_{k,g} * \mathbf{x} + \underline{V_{k,g}^T \mathbf{h}})$

- They modeled the conditional distribution by adding terms that depend on hto the activations before the nolinearities

■ PixelCNN AutoEncoder

- From a convolutional auto-encoder, they replacedthe deconvolutional decoder with conditional PixelCNN



Experimental Results (Unconditional)

- Score: Negative log-likelihood score (bits/dim)

- Data: CIFAR-10 dataset

Model	NLL Test (Train)
Uniform Distribution: [30]	8.00
Multivariate Gaussian: [30]	4.70
NICE: [4]	4.48
Deep Diffusion: [24]	4.20
DRAW: [9]	4.13
Deep GMMs: [31, 29]	4.00
Conv DRAW: [8]	3.58 (3.57)
RIDE: [26, 30]	3.47
PixelCNN: [30]	3.14 (3.08)
PixelRNN: [30]	3.00 (2.93)
Gated PixelCNN:	3.03 (2.90)

- Gated PixelCNN outperforms the PixelCNN by 0.11 bits/dim, which has a very significant effect on the visual quality, and close to the performance of PixelRNN

- Data: ImageNet dataset

32x32	Model	NLL Test (Train)
	Conv Draw: [8]	4.40 (4.35)
	PixelRNN: [30]	3.86 (3.83)
	Gated PixelCNN:	3.83 (3.77)
64x64	**Model**	**NLL Test (Train)**
	Conv Draw: [8]	4.10 (4.04)
	PixelRNN: [30]	3.63 (3.57)
	Gated PixelCNN:	3.57 (3.48)

- Gated PixelCNN outperforms PixelRNN.
- Achieve similar performance to the PixelRNN in less than half the training time.

Experimental Results

■ Coniditioning on ImageNet classes

- Given a one-hot encoding h_i, for the i-th class, model $p(x \mid h_i)$

Lhasa Apso (dog)

Sorrel horse

(part of results.)

■ Coniditioning on Portrait Embeddings

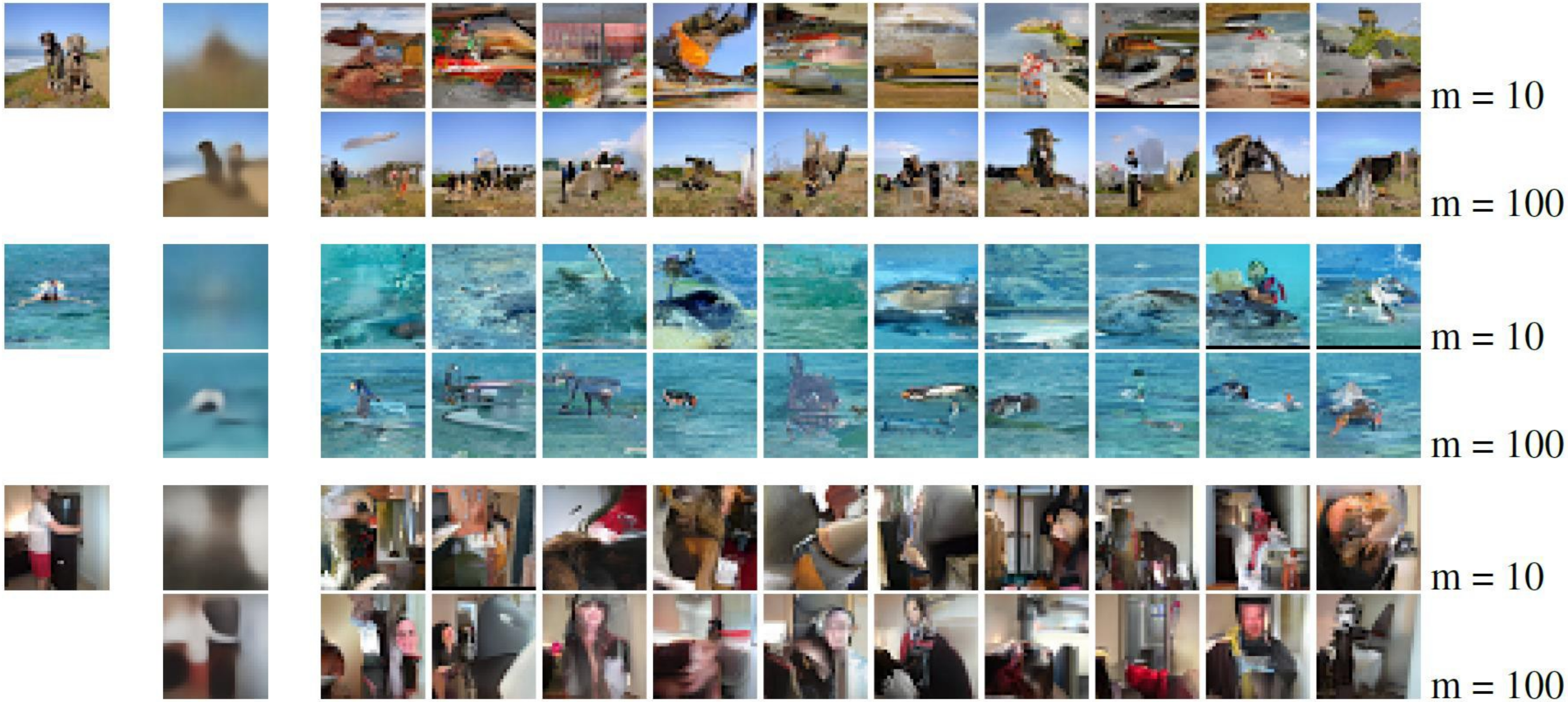
- Embeddings are took from top layer of a conv network trained on a large database of portraits from Flickr images.
- After the supervised net was trained, {x:image, h:embedding} tuples are taken and trained conditional PixelCNN to model $p(x \mid h)$
- Given a new image of a person that was not in the training set, they computed h and generate new portraits of same person.

- And experimented with reconstructions conditioned on linear interpolations between embeddingsof pairs of images.

Experimental Results (PixelCNN Auto-Encoder)

- Data: 32x32 ImageNet patches

(m: dimensional bottleneck)



(Left to right: original image, reconstruction by auto-encoder, conditional samples from PixelCNN auto-encoder)

- Assumption of Autoregressive Models
- Fully visible Sigmoid Belief Network (FVSBN)
- Neural Autoregressive Density Estimation (NADE)
- Real-valued Neural Autoregressive Density Estimation (RNADE)
- Autoregressive Autoencoders
- Masked Autoencoder for Distribution Estimation (MADE)
- Recurrent Neural Networks
- Pixel RNN: Pixel CNN, Row LSTM and Diagonal BiLSTM
- Gated PixelCNN
- WaveNet

Motivation

The Autoregressive model (e.g. PixelCNN) has been very successful.

→ What about voice?

I want to do that with a CNN which is more efficient than an RNN.

WaveNet

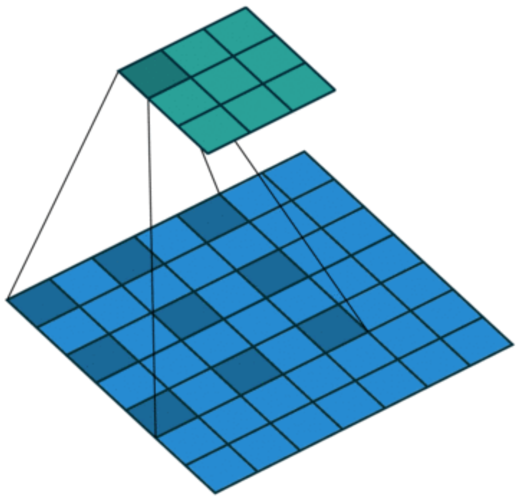
This network is similar to PixelCNN.

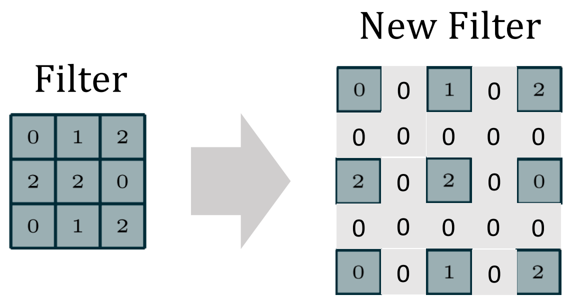
Here are the contributions:

- Unprecedented quality speech synthesis.
- Efficient architecture with a large receptive field using dilated convolution
- (also voice recognition)

Recap: Dilated convolution

Roughly speaking, if you really want to use a filter with a large kernel size, you can use this to get a large kernel and approximate results without increasing the amount of computation.



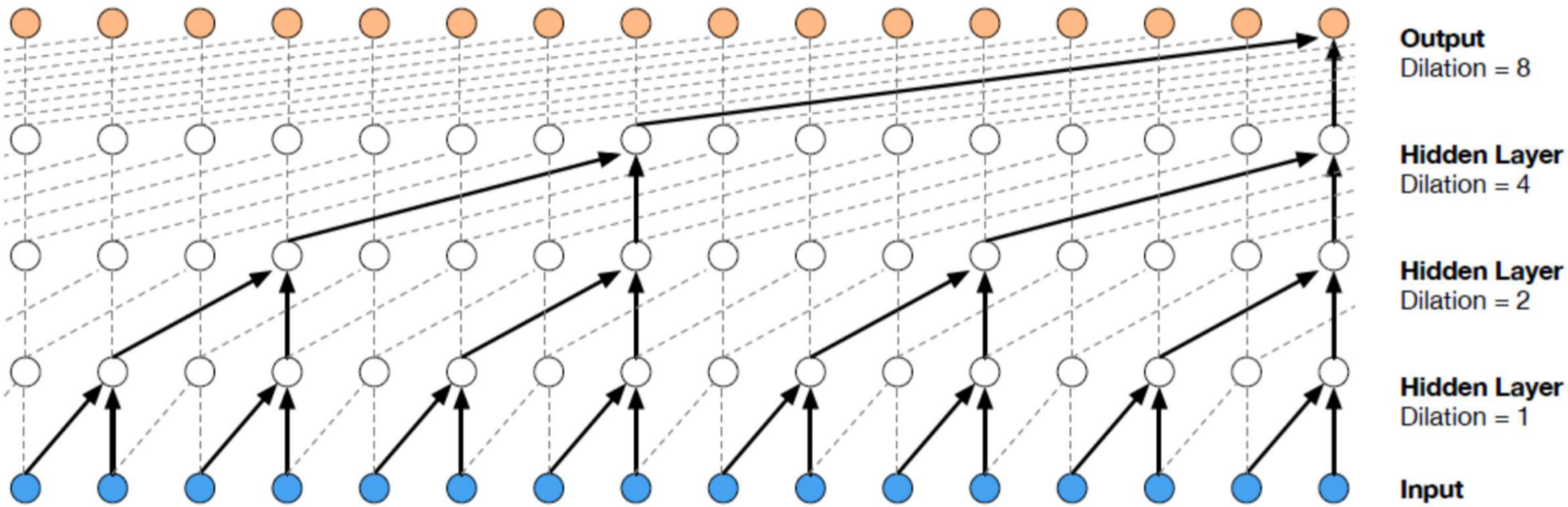


Dilation rate = 2×2

Receptive field : 3×3 → 5×5

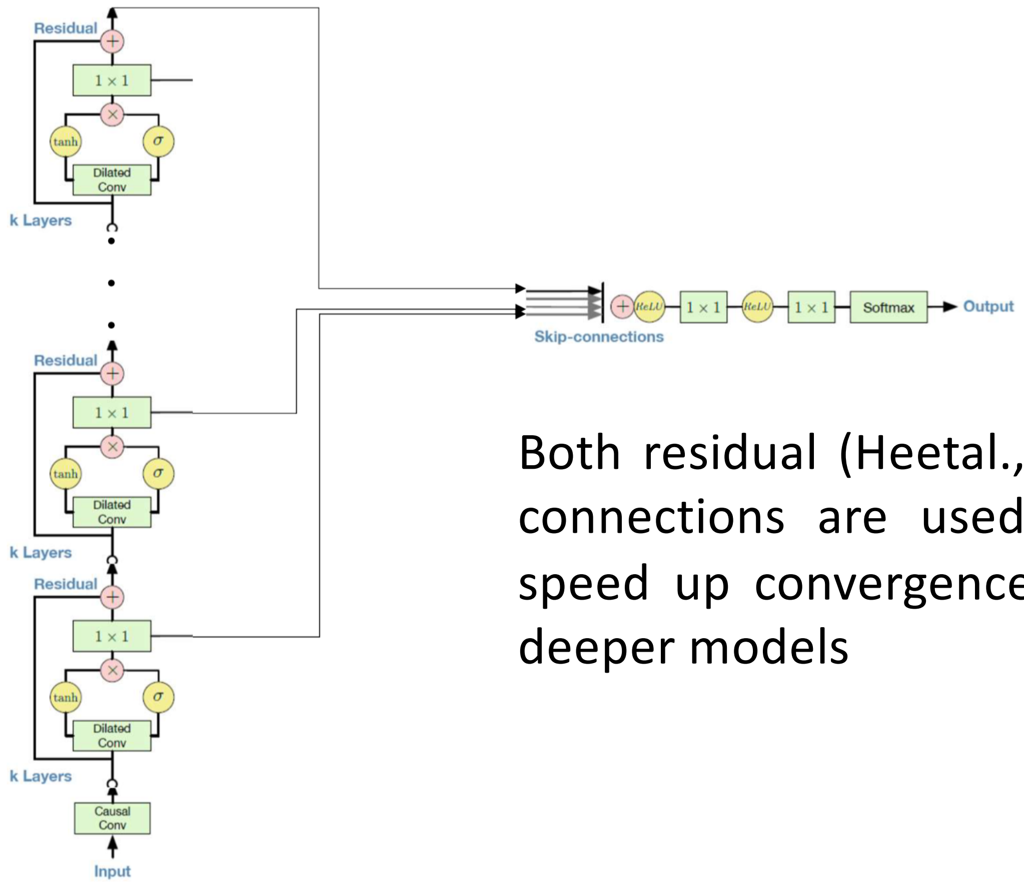
Stacked dilated causal convolution

This is a conceptual diagram of the expansion of the receptive field.



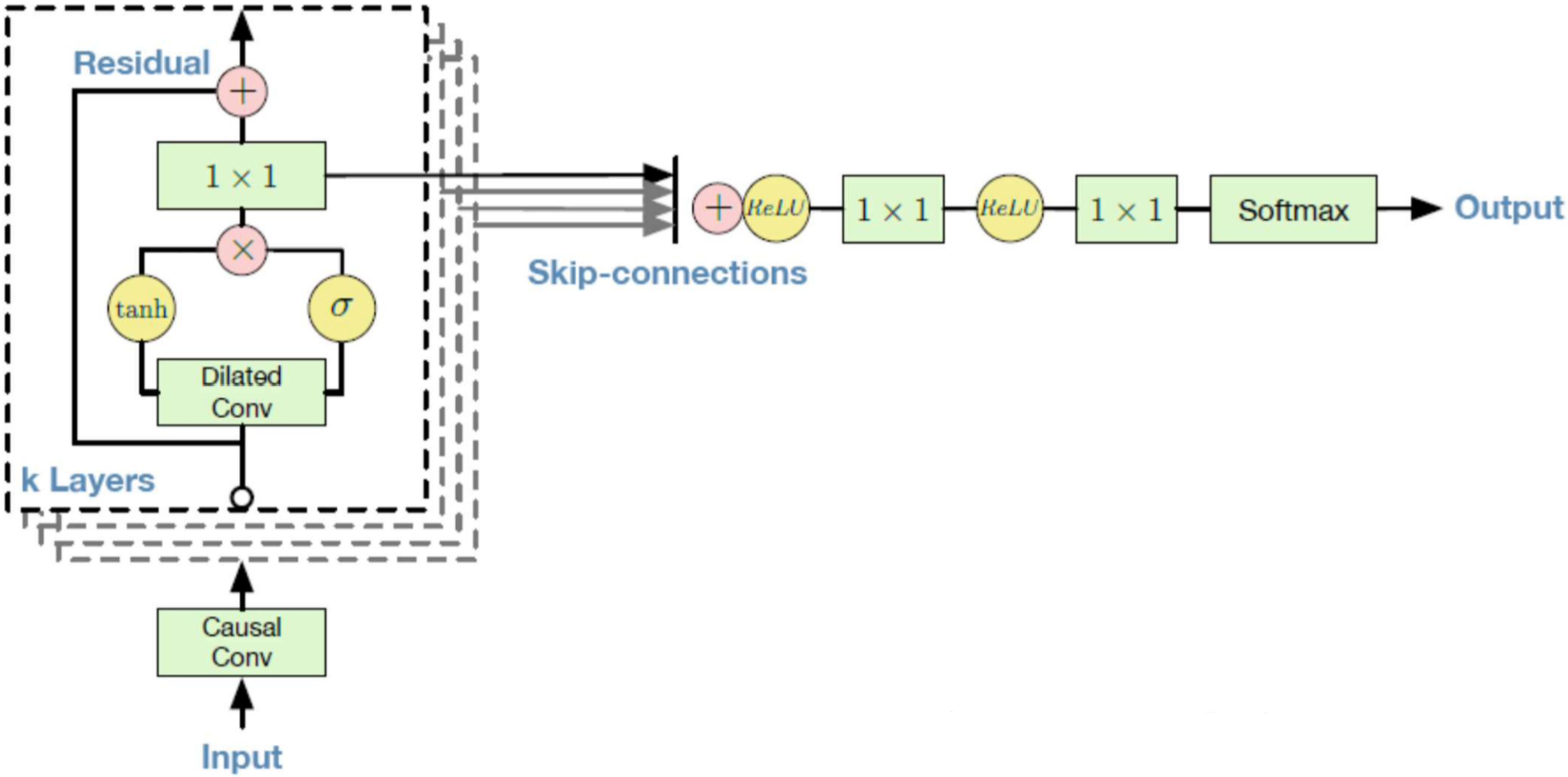
Entire architecture

PEKING UNIVERSITY



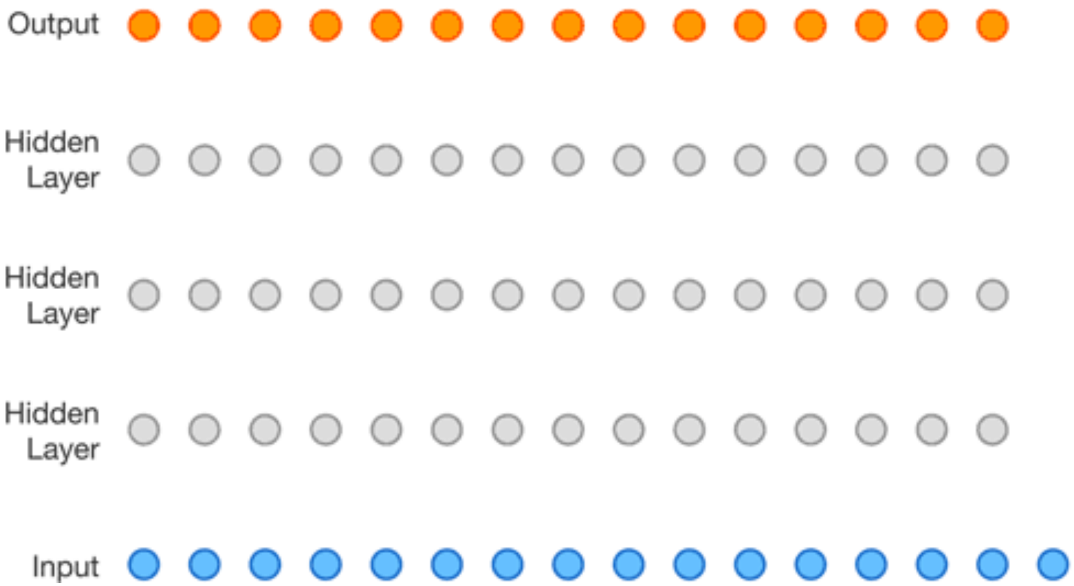
Both residual (Heetal.,2015) and parameterized skip-connections are used throughout the network, to speed up convergence and enable training of much deeper models

Residual block



Conditional Generation

$$P(X = speech|Y = sentence)$$



Parametric

Concatenative

WaveNet

Unconditional

Music

WaveNet: A Generative Model for Raw Audio. *A. Oord, S. Dieleman et al. arXiv. 2016.*

Summary of Autoregressive Models

Easy to sample from

Sample $x_0 \sim p(x_0)$

Sample $x_1 \sim p(x_1 \mid x_0 = x_0)$

-

Easy to compute probability $p(x = x)$

Compute $p(x_0 = x_0)$

Compute $p(x_1 = x_1 \mid x_0 = x_0)$

Multiply together (sum their logarithms)

-

Ideally, can compute all these terms in parallel for fast training

Easy to extend to continuous variables. For example, can choose Gaussian conditionals $p(x_t \mid x_{<t}) = N(\mu_\vartheta(x_{<t}), \Sigma_\vartheta(x_{<t}))$ or mixture of logistics

No natural way to get features, cluster points, do unsupervised learning

References

Pixel Recurrent Neural Networks:
https://arxiv.org/pdf/1609.03499.pdf

WaveNet: A Generative Model for Raw Audio:

https://arxiv.org/pdf/1601.06759.pdf

Conditional Image Generation with PixelCNN Decoders:

https://arxiv.org/pdf/1606.05328v2.pdf

Thanks