Energy-based Models

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Content

- Energy-based models
  - Why not probabilistic models?
  - Introduction
  - Training and inference
- Some works
  - Deep Belief Network
  - EBGAN
  - BEGAN
  - MAGAN

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Likelihood based learning

- Main concern: probability distributions $p(x)$
  - Non-negative: $p(x) \geq 0$
  - Sum-to-one: $\sum_x p(x) = 1 \; or \; \int p(x)dx = 1$

- Non-negative is easy
  - $f^2, \exp(f)$,…, where f is any neural network

- Sum-to-one is important
  - Increasing $p(x_{train})$ means x_{train} is more likely than others
  - Difficult to realise

Likelihood based learning

- Sum-to-one:
  - Some functions are easy to normalized analytically
    - Exponential: $f_\lambda(x) = e^{-\lambda x}$, $\int f_\lambda(x)dx = \frac{1}{\lambda}$
    - Gaussian: $\mathrm{f}(x) = e^{\frac{(x-\mu)^2}{2\sigma^2}}$, $\int f(x)dx = \sqrt{2\pi\sigma^2}$
  - Some models can be obtained by combining these functions
    - Autoregressive: products of normalized objects
      - $\iint_{xy} p_\theta(x)p_{\theta'(x)}(y)\,dxdy = 1$
    - Latent variables: Mixtures of normalized objects
      - $\int \alpha p_\theta(x) + (1-\alpha)p_{\theta'}(x)dx = 1$
  - But other functions are difficult to compute analytically

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Energy based model

- $p_\theta(x) = \frac{\exp\left(-E_\theta(x)\right)}{\int \exp\left(-E_\theta(x)\right)dx} = \frac{\exp\left(-E_\theta(x)\right)}{Z(\theta)}$
  - $E_\theta(x)$ is called energy function
  - $Z(\theta) = \int \exp\left(-E_\theta(x)\right) dx$ is called partition function
  - Gibbs/Boltzmann Distribution
- Why this format?
  - Exponential and log are the natural scale
    - Pretty much functions can be rewritten in this format
    - In accordance with statistical physics
  - MCMC + Langevin equation

Energy versus Probabilistic

- $p_\theta(x) = \frac{\exp\left(-E_\theta(x)\right)}{\int \exp\left(-E_\theta(x)\right)dx} = \frac{\exp\left(-E_\theta(x)\right)}{Z(\theta)}$

- Why not probabilistic approaches?
  - Partition function problem
    - High probability for good answers
    - Low probability for bad answers
    - Too many bad answers!

Energy-based model

- Pros:
  - Flexibility: use pretty much functions as energy functions
  - A unified framework for all these probabilistic and non-probabilistic approaches
  - Normalization is not required sometimes
- Cons:
  - Sampling from p(x) is difficult
  - Learning process is hard
  - Features are not learned (but can add latent variables)
  - Energies are uncalibrated

Energy-based model

- $p_\theta(x) = \frac{\exp\left(-E_\theta(x)\right)}{\int \exp\left(-E_\theta(x)\right)dx} = \frac{\exp\left(-E_\theta(x)\right)}{Z(\theta)}$

- Curse of dimensionality
  - Computing $Z(\theta)$ numerically (when there's no analytic solution) scales exponentially in the number of dimensions of x.
  - Some tasks do not require knowing $Z(\theta)$

Energy-based model

- $p_\theta(x) = \frac{\exp(-E_\theta(x))}{\int \exp(-E_\theta(x))dx} = \frac{\exp(-E_\theta(x))}{Z(\theta)}$
- Given x, x', evaluating $p_\theta(x), p_\theta(x')$ is hard because of Z
- However, their ratio is easy to obtain
  - $\frac{p_\theta(x)}{p_\theta(x\prime)} = \exp(f_\theta(x) - f_\theta(x'))$

What Questions can a model answer?

- 1. Classification & Decision Making:
  - Which value of Y is most compatible with X?
  - Application: Robot navigation, …
  - Training: give the lowest energy to the correct answer

- 2. Ranking:
  - Is Y1 or Y2 more compatible with X?
  - Applications: Data-mining, …
  - Training: produce energies that rank the answers correctly

What Questions can a model answer?

- 3. Detection:
  - Is this value of Y compatible with X?
  - Application: face detection, …
  - Training: energies that increase as the image looks less like a face

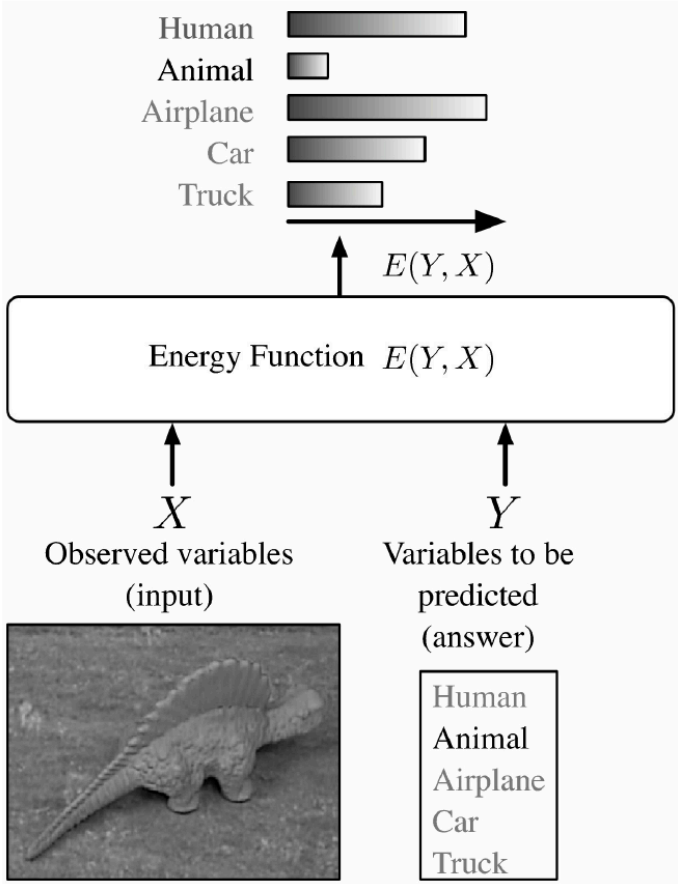
- 4. Conditional Density Estimation:
  - What is the conditional distribution P(Y|X)?
  - Applications: decision-making system, …
  - Training: differences of energies must be just so.

What Questions can a model answer?

- 5. Generative models:
  - What is the generative results Y of X?
  - Application: denoising, completion, generation, …
  - Training: lower energies to better answer

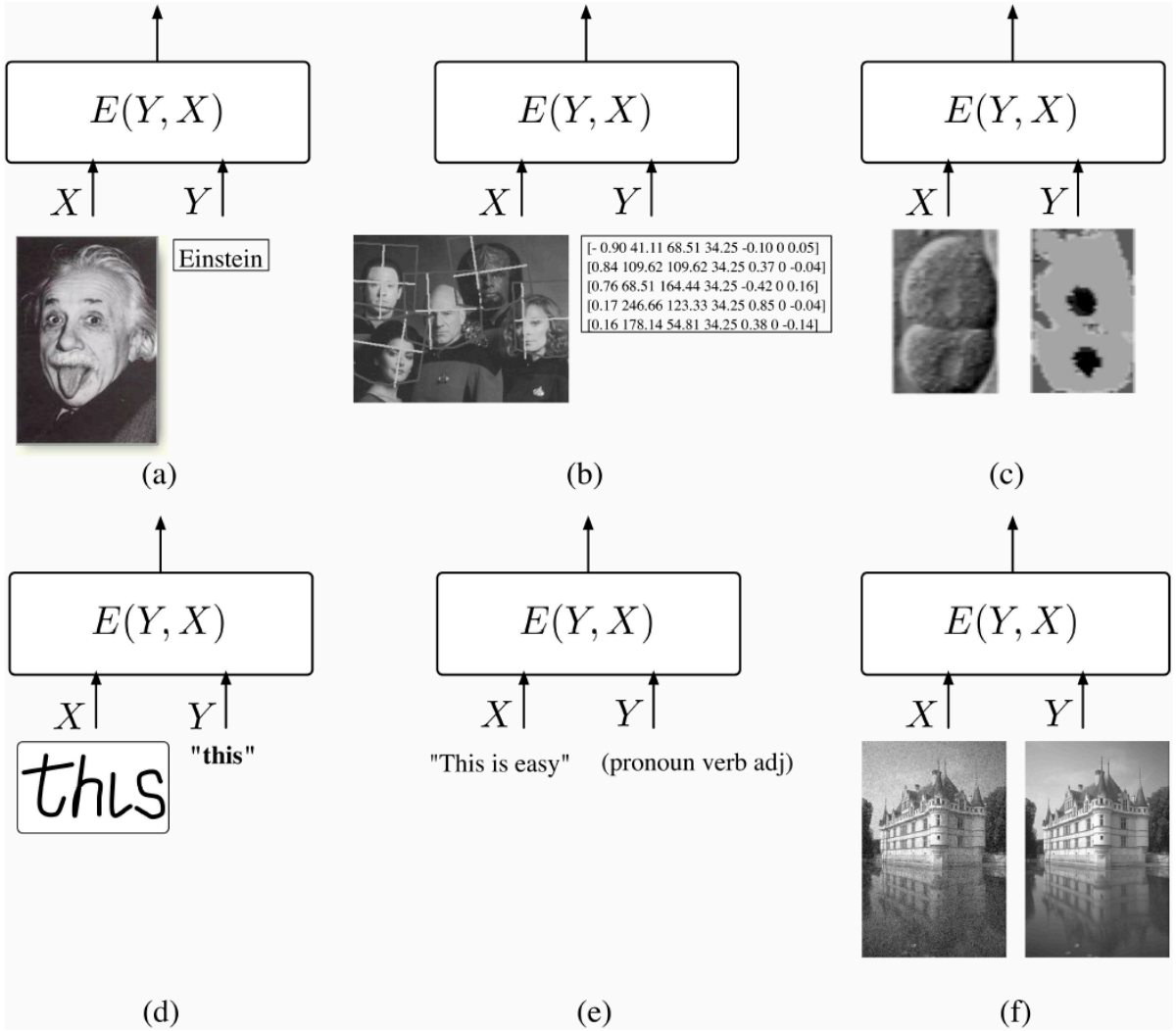
Energy-based model for decision-making

- Model:
  - measures the compatibility between an observed variable X and a variable to be predicted Y through an energy function E(Y, X)
- Inference:
  - Search for Y that minimize the energy within a set $\mathcal{Y}$
  - Low cardinality: exhaustive search



Energy-based model for decision-making

- Inference:
  - Search for Y that minimize the energy within a set $\mathcal{Y}$
  - High cardinality: min-sum, Viterbi, …



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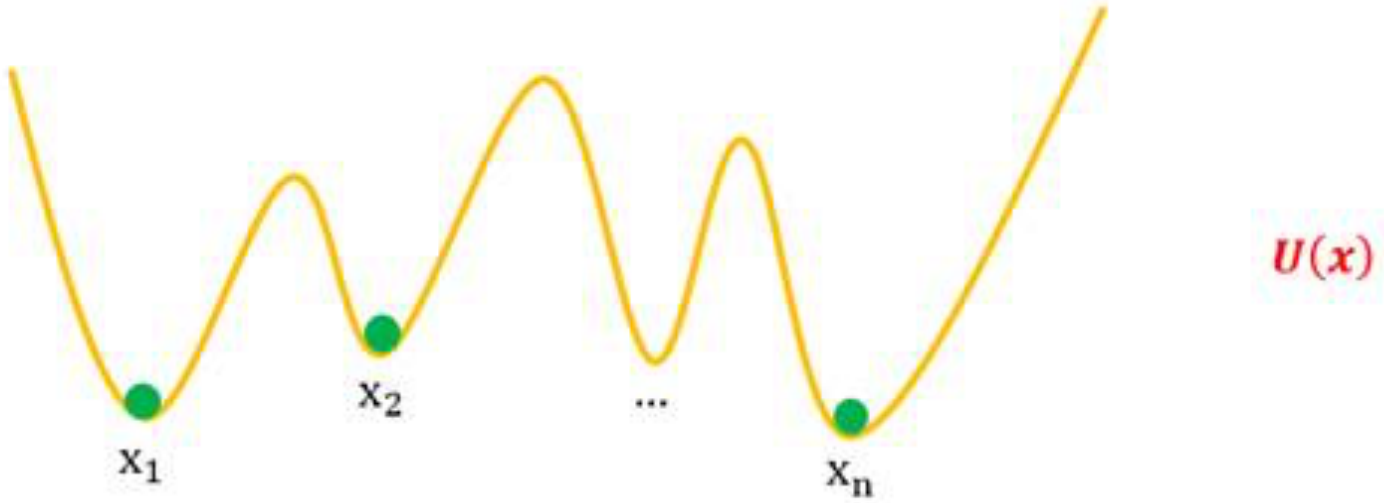
Training Intuition

- A random weight at first
  - The energy is a line

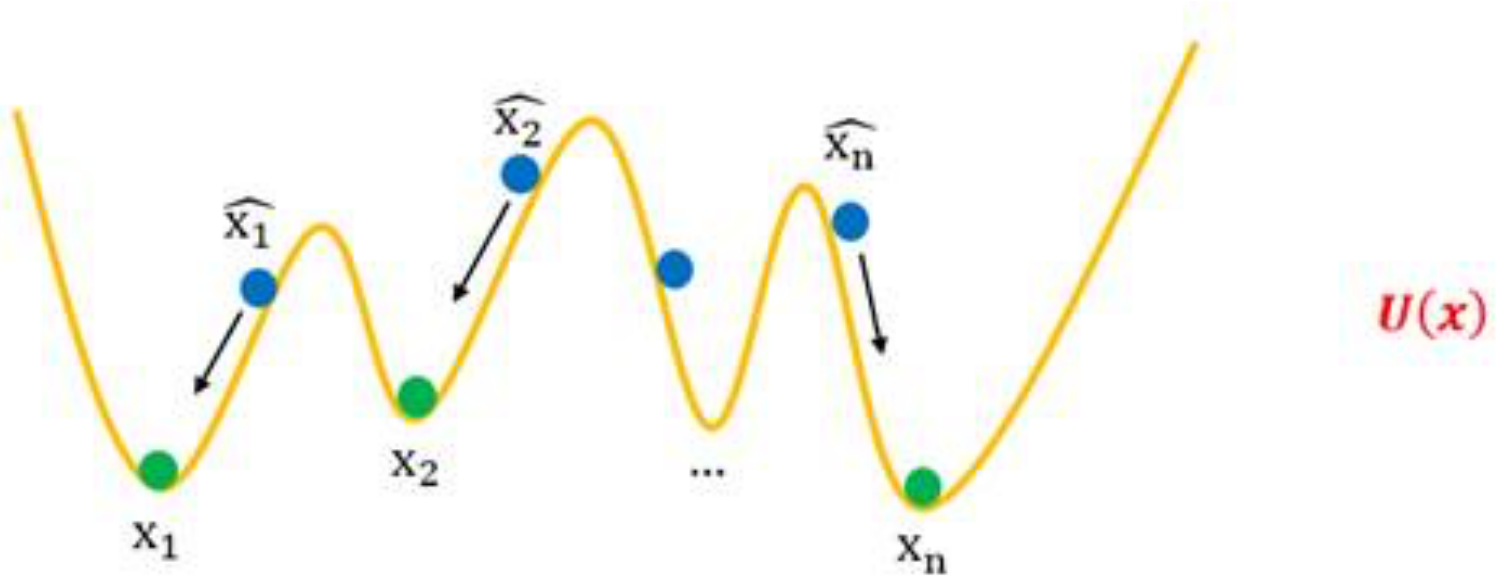
Training Intuition

- Real samples should be the valley
- Fake samples should be high (if exist)



Inference Intuition

- Samples will slide to the valley



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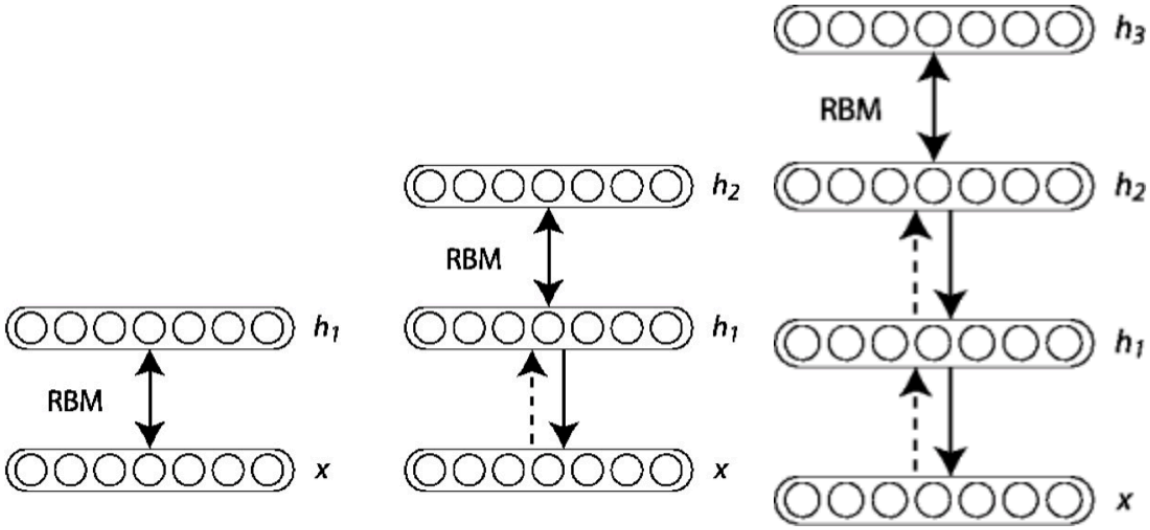
Deep Belief Network

- Problem of multi-layer neural network
  - The gradients may be too large or small

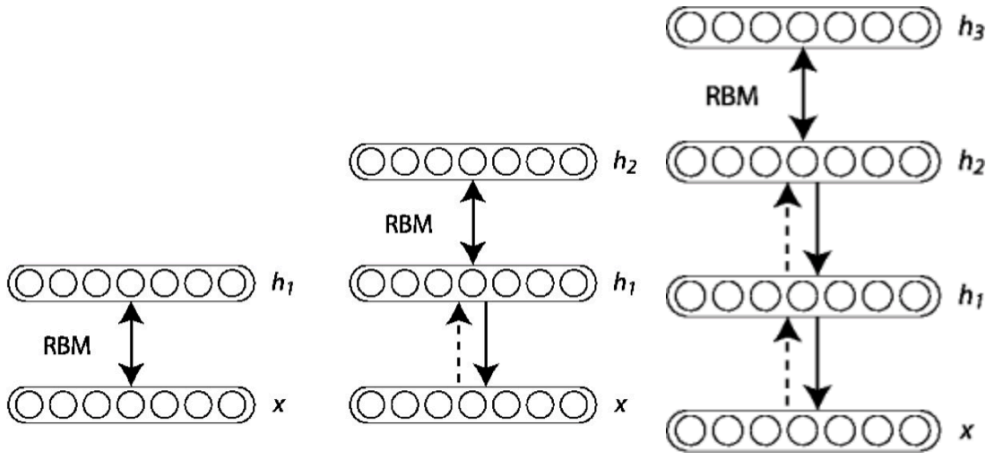
- What if the initial value is close to the optimal value?

- Deep Belief Network proposed by Hinton in 2006



Deep Belief Network

- Training Process:
  - View x and h1 as a RBM1 and train the weights
  - Fix the weights for RBM1, and train RBM2 (visible units: h1, hidden units: h2)
  - …
  - For the last layer, output what we want, calculate the difference and update weights



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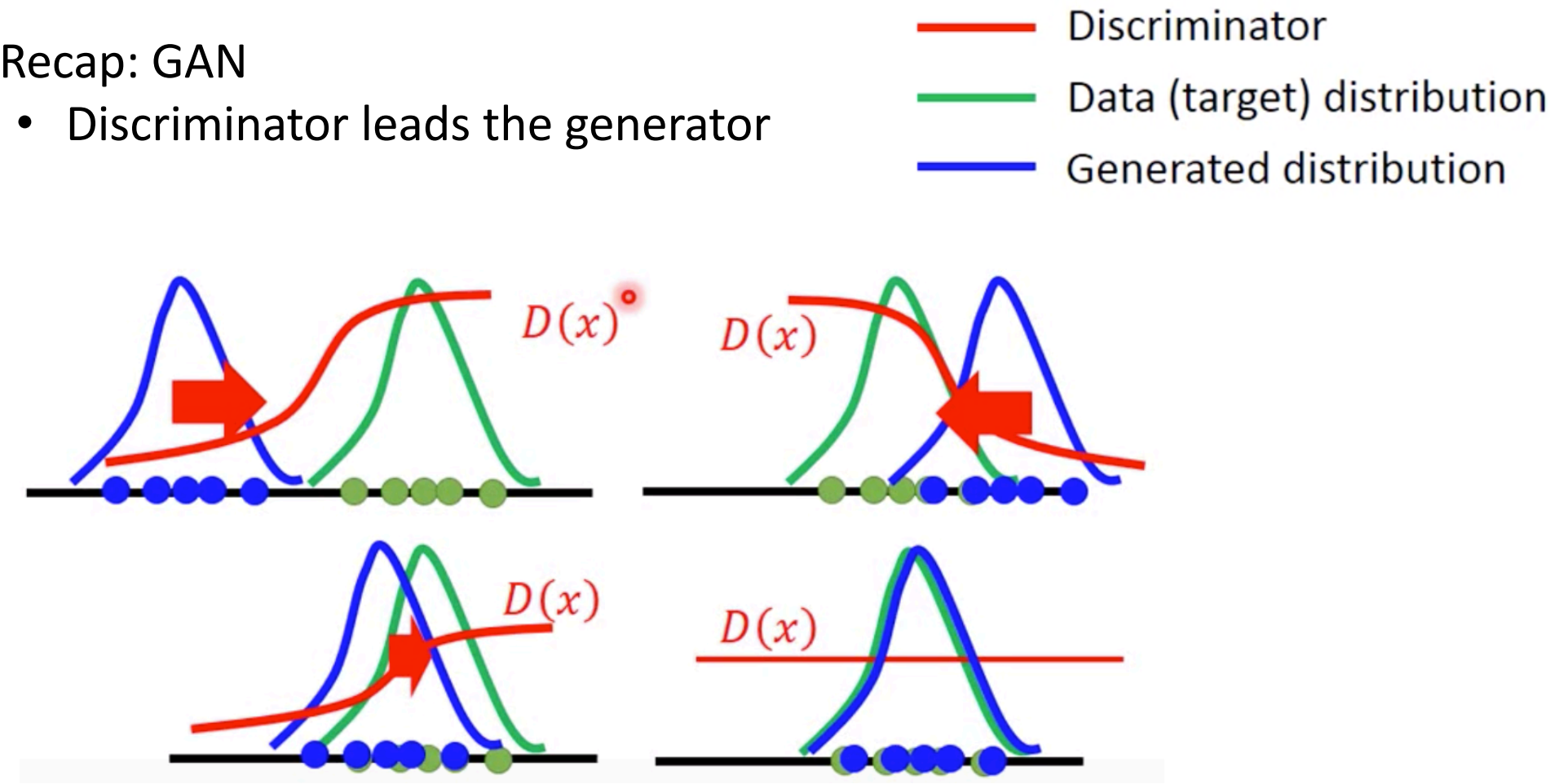
Energy-based GAN (EBGAN)

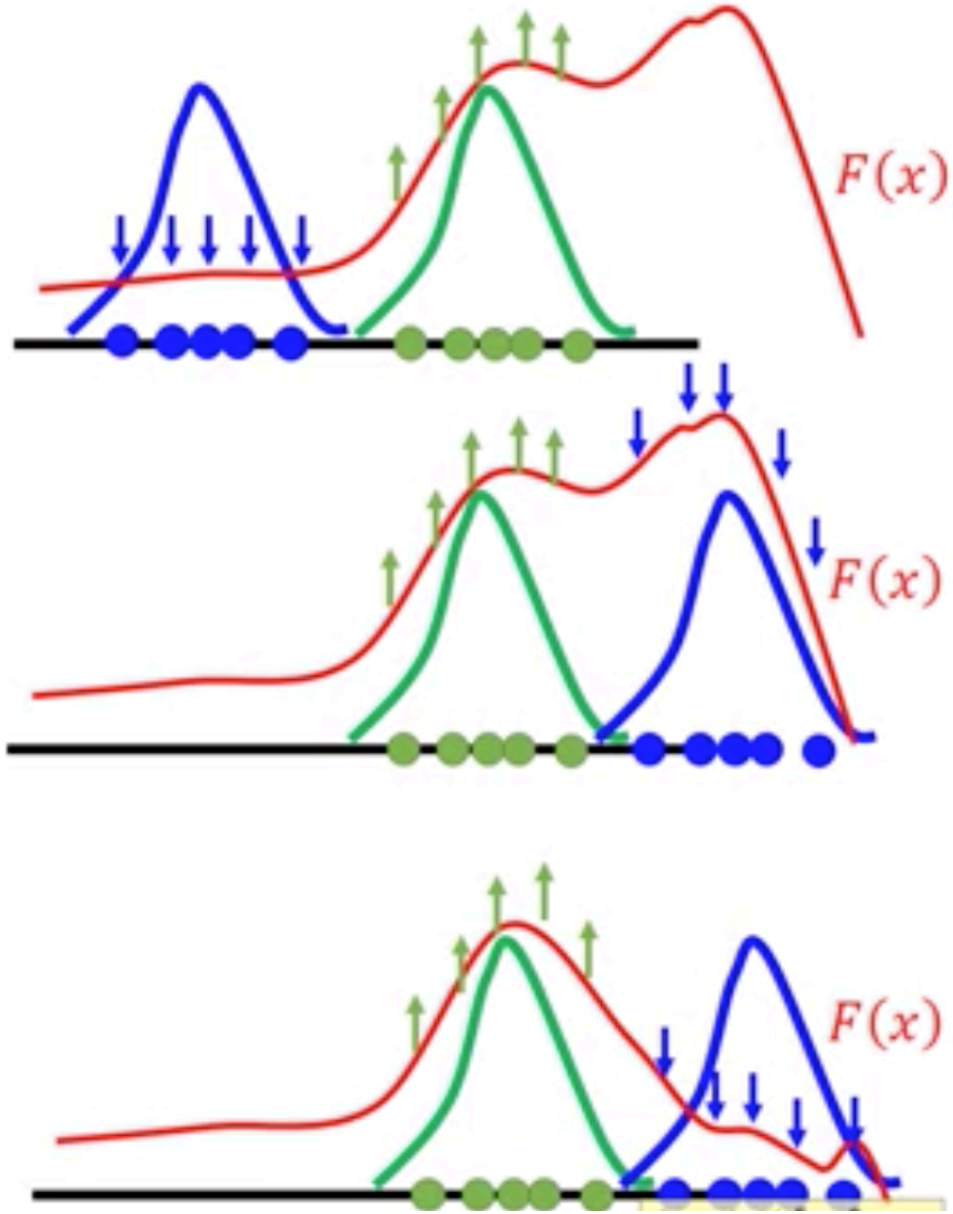
- Recap: GAN
  - Discriminator leads the generator



Energy-based GAN (EBGAN)

- Recap:
  - We want the energy of positive examples to be low
  - The energy of negative examples to be high
  - But it's difficult to update for all negative examples

- Generator is an intelligent way to find the negative examples
- F is the Discriminator

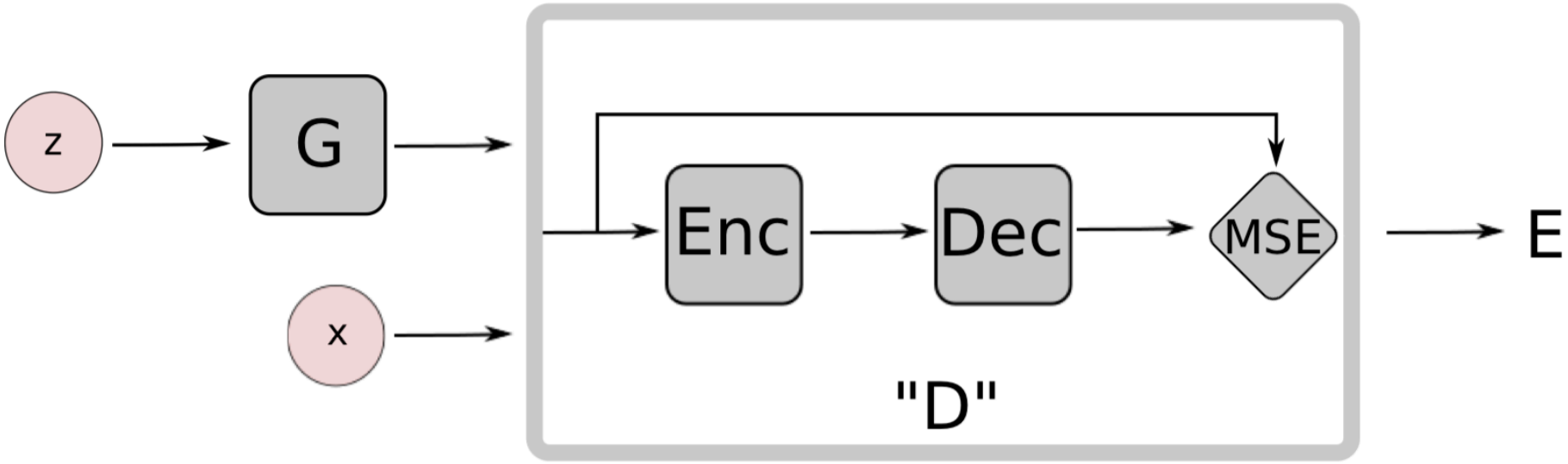


Energy-based GAN (EBGAN)

- View the discriminator as an energy function
- Auto-encoder as discriminator
- Loss function with margin for discriminator training

- Results:
  - Able to generate high-quality 256*256 images

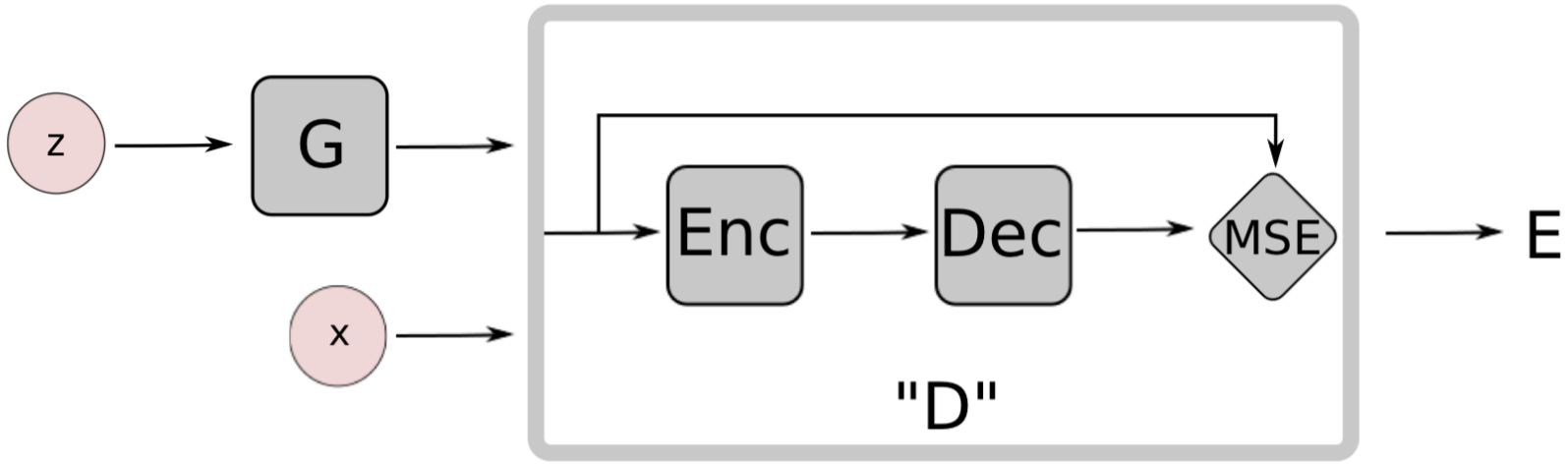
Energy-based GAN (EBGAN)



$$D(x) = ||Dec(Enc(x)) - x||.$$

- Real examples: D(x) → 0
- Fake example: D(x) should be large

Training Process

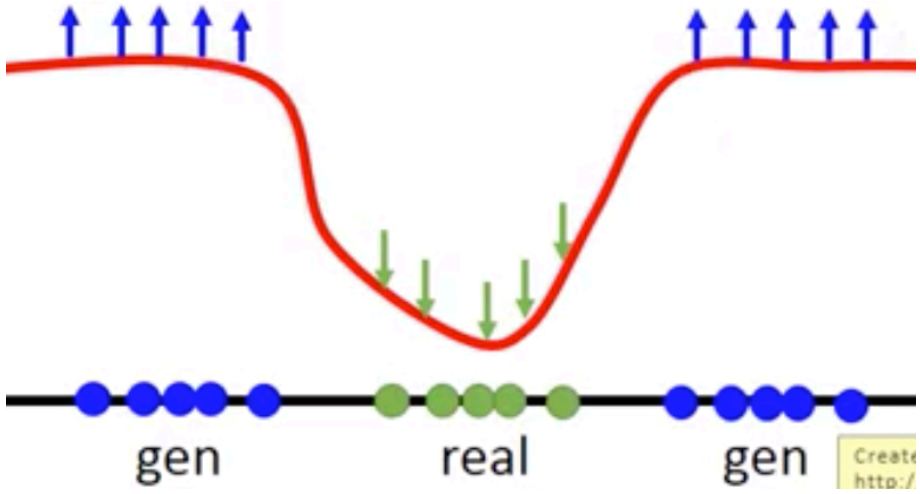


- Sample real example x
- Sample code z for prior distribution
- Update discriminator D to minimize
  - $L_D(x, z) = D(x) + \max(0, m - D(G(z)))$
- Sample code z from prior distribution
- Update generator G to minimize
  - $L_G(z) = D(G(z))$

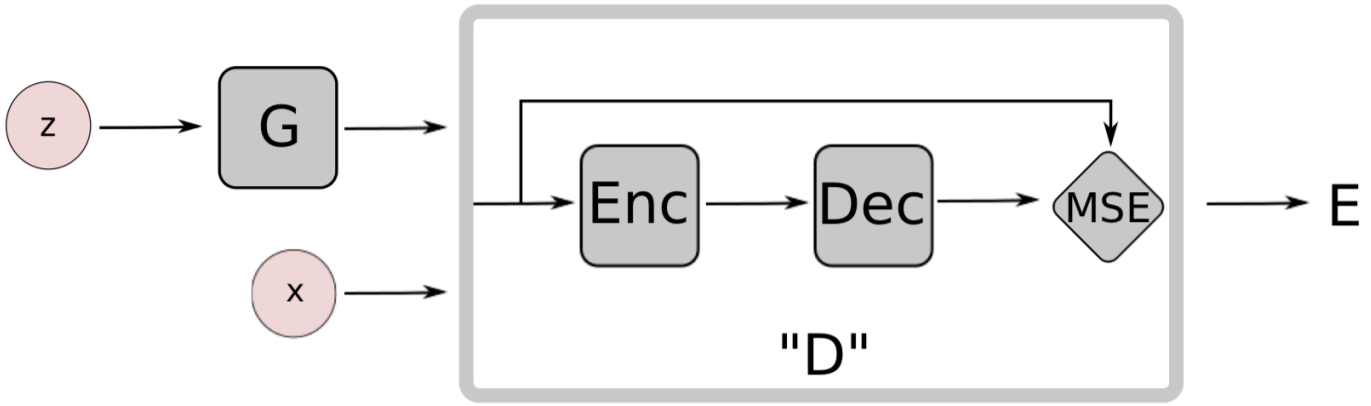
Energy-based GAN (EBGAN)

- Why $L_D(x, z) = D(x) + \max(0, m - D(G(z)))$
- But not $L_D(x, z) = D(x) - D(G(z))$?

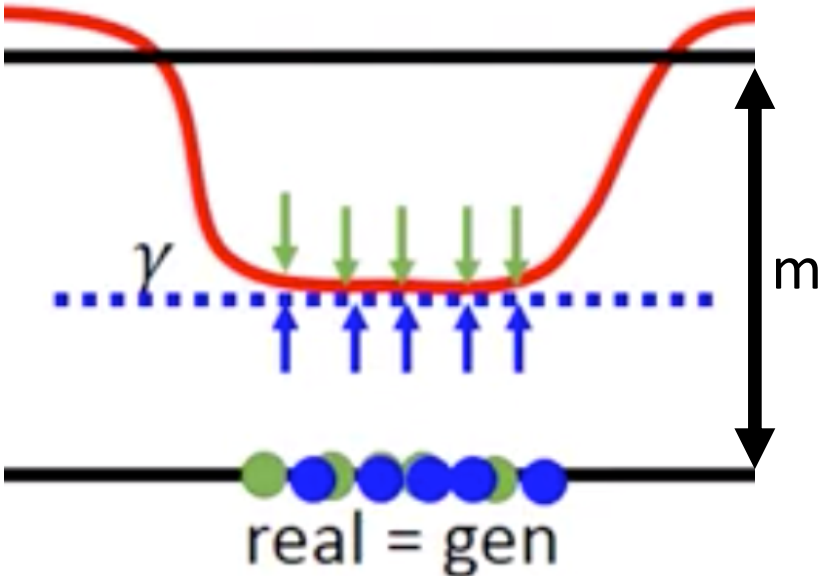
- D(fake) can be infinite large
- So D will not focus on real example



Energy-based GAN (EBGAN)

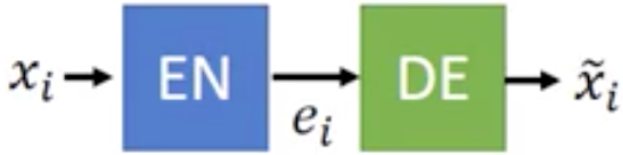


- Finally, D(real) and D(gen) will be $\gamma \in (0, m)$



Energy-based GAN (EBGAN)

- Pulling-away term for training generator
  - For diverse outputs
  - Given a batch outputs of generator $S = \{x_1, \dots, x_N\}$
  - $f_{PT}(S) = \sum_{i,j,i \neq j} \cos(e_i, e_j)$



- Better way to learn auto-encoder
  - If only minimize the recon error of real images: lead to identity function
  - Giving large reconstruction error for fake images regularized auto-encoder

Energy-based GAN (EBGAN)

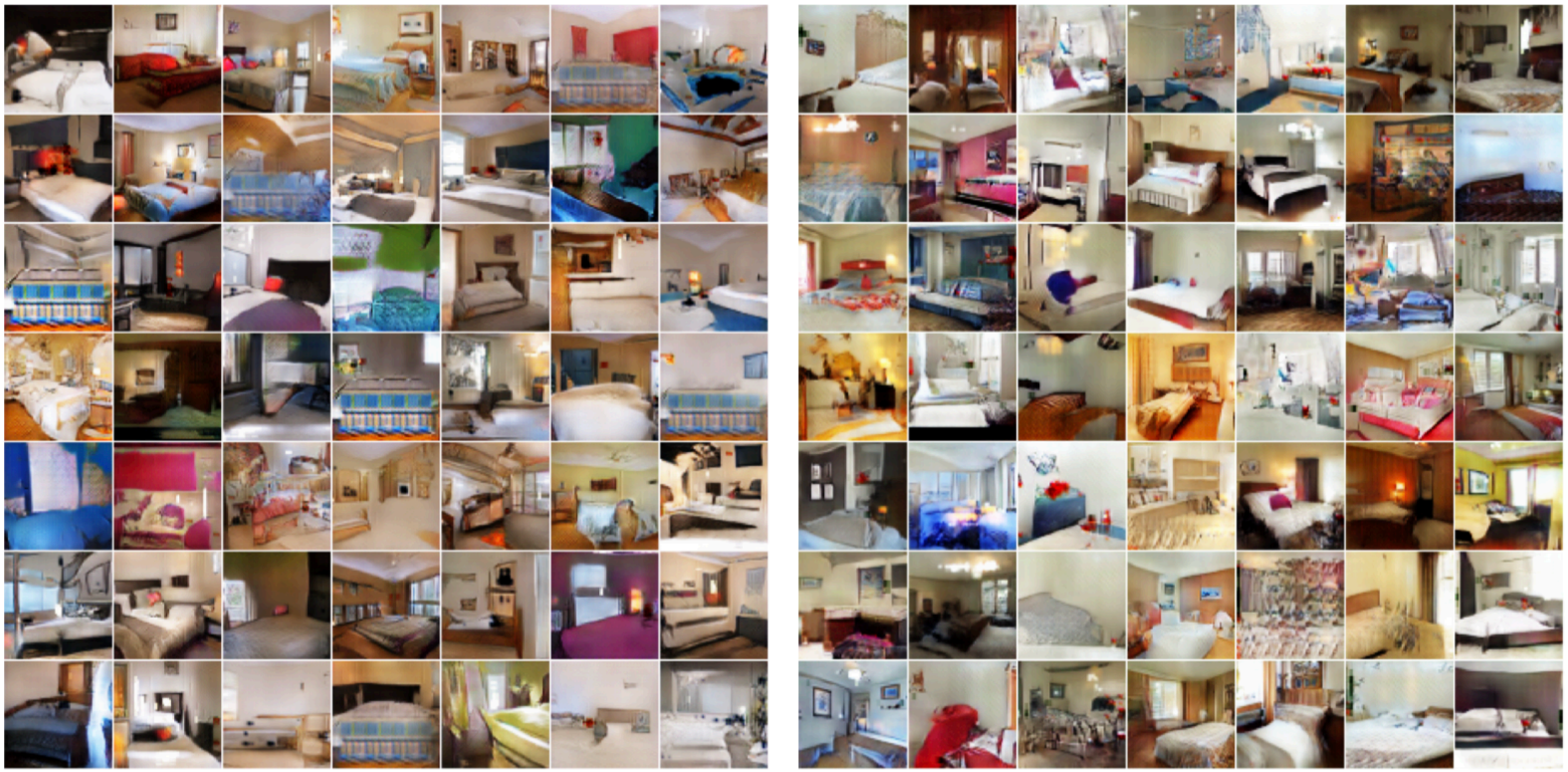


Figure 5: Generation from the LSUN bedroom dataset. Left(a): DCGAN generation. Right(b): EBGAN-PT generation.

Energy-based GAN (EBGAN)

Figure 8: ImageNet 256×256 generations using an EBGAN-PT.

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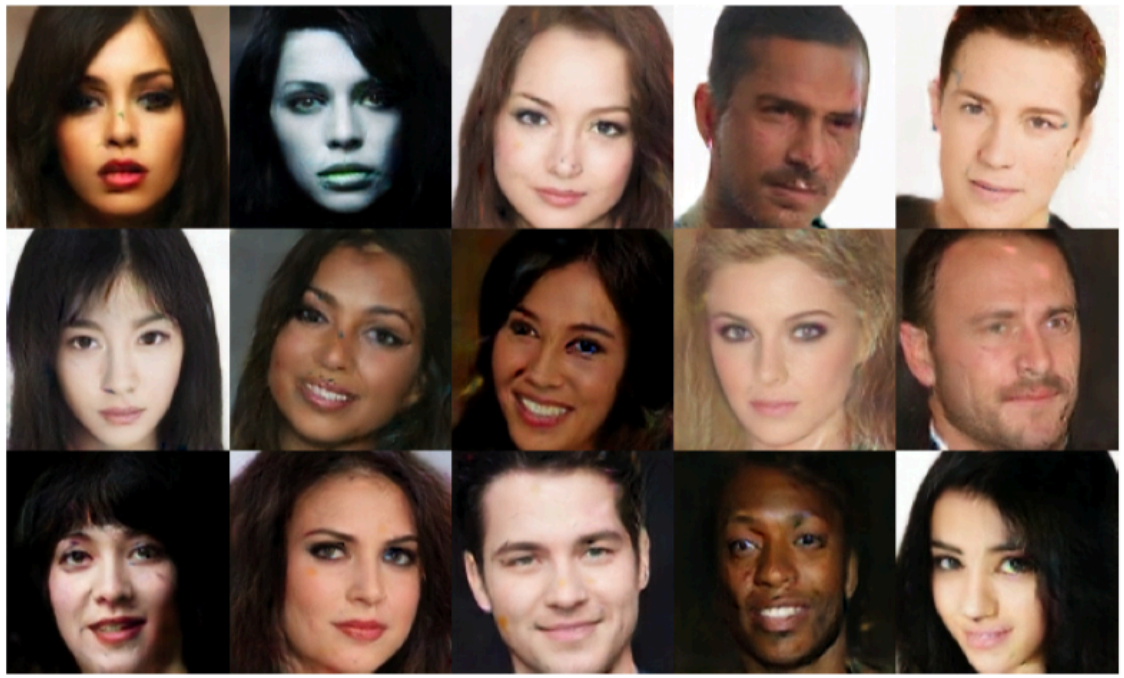
Boundary Equilibrium GAN (BEGAN)

$$\begin{cases} \mathcal{L}_D = \mathcal{L}(x) - k_t.\mathcal{L}(G(z_D)) & \text{for } \theta_D \\ \mathcal{L}_G = \mathcal{L}(G(z_G)) & \text{for } \theta_G \\ k_{t+1} = k_t + \lambda_k(\gamma\mathcal{L}(x) - \mathcal{L}(G(z_G))) & \text{for each training step } t \end{cases}$$

- Auto-encoder based GAN
- K0 = 0
- Increase when : $\gamma L(x) > L(G(z_G))$

Boundary Equilibrium GAN (BEGAN)

$$\begin{cases} \mathcal{L}_D = \mathcal{L}(x) - k_t.\mathcal{L}(G(z_D)) & \text{for } \theta_D \\ \mathcal{L}_G = \mathcal{L}(G(z_G)) & \text{for } \theta_G \\ k_{t+1} = k_t + \lambda_k(\gamma\mathcal{L}(x) - \mathcal{L}(G(z_G))) & \text{for each training step } t \end{cases}$$



Boundary Equilibrium GAN (BEGAN)

$$\begin{cases} \mathcal{L}_D = \mathcal{L}(x) - k_t.\mathcal{L}(G(z_D)) & \text{for } \theta_D \\ \mathcal{L}_G = \mathcal{L}(G(z_G)) & \text{for } \theta_G \\ k_{t+1} = k_t + \lambda_k(\gamma\mathcal{L}(x) - \mathcal{L}(G(z_G))) & \text{for each training step } t \end{cases}$$

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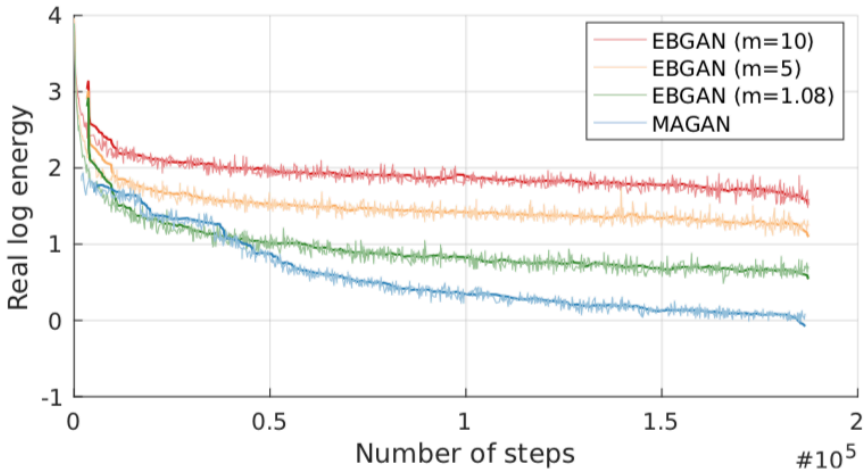
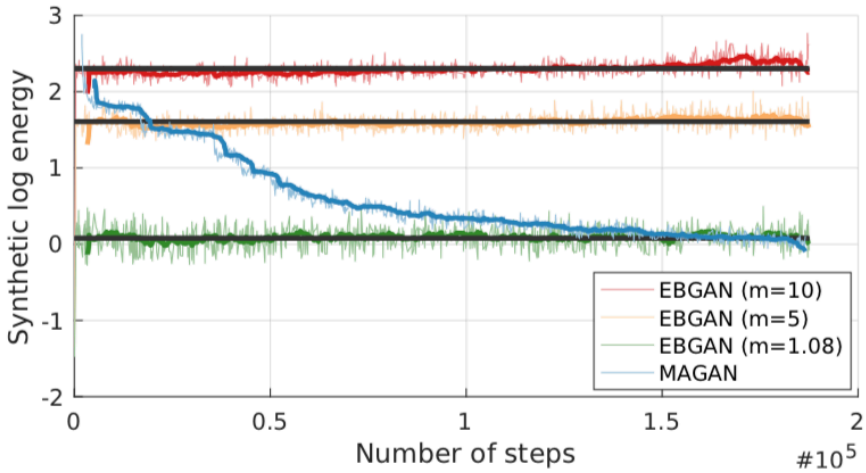
Margin Adaptation GAN (MAGAN)

- Dynamic margin m
  - As the generator generates better images
  - The margin becomes smaller if satisfies the conditions
  - Three conditions: $E_G^{t-1} \leq E_G^t$ and $E_{data}^t < m_t$ and $E_{data}^t < E_G^t$

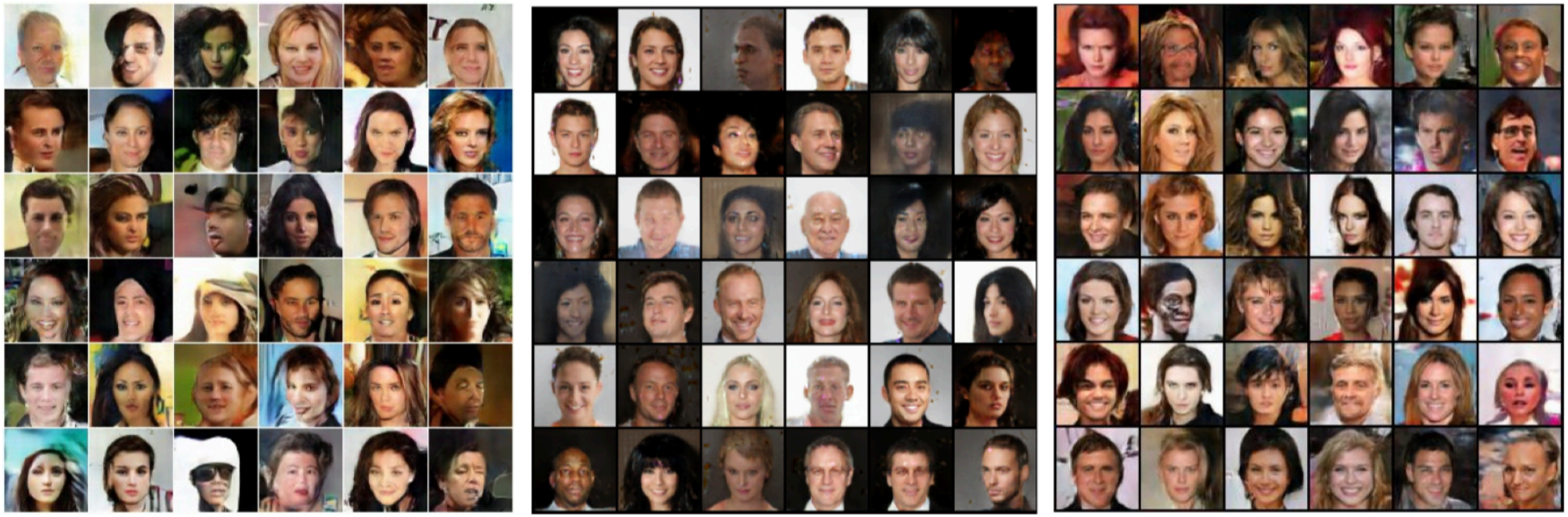


(a) Comparison of real samples energy between proposed method and EBGAN



(b) Comparison of synthetic samples energy between proposed method and EBGAN

Margin Adaptation GAN (MAGAN)



(a) (b) (c)

Figure 2: (a) EBGANs CelebA generation taken from [8]. (b) BEGANs CelebA generation based on [21]. (c) CelebA generation from our method. Results from BEGANs and our method are from a random mini-batch of generates samples respectively. Best viewed in color and enlarged. More samples are available in the Supplementary Material.

Reference

- LeCun et. al, A Tutorial on Energy-Based Learning
- Stanford CS 236 Lecture 11
- Energy-based GAN, Hung-yi Lee

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