Normalizing Flow Models (Part 2)

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So far

- Learning via **maximum likelihood** over the dataset D

$$\max_{\theta} logp(D;\theta) = \sum_{x \in D} log\pi\left(G_{\theta}^{-1}(x)\right) + log\left|det\left(\frac{\partial G_{\theta}^{-1}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)\right|$$

inverted function　　determinant of Jacobian

- What we need?
  - prior $z \sim \pi(z)$ easy to sample
  - Invertible transformations
  - Determinants of Jacobian Efficient to compute

Reference slides

- Hung-yi Li. Flow-based Generative Model
- Stanford “Deep Generative Models”. Normalizing Flow Models

- Coupling layer based normalizing flow models
  - Coupling layer
  - NICE
  - Real NVP
  - Glow
- Autoregressive models as flow models
  - MAF
  - IAF
  - Parallel Wavenet

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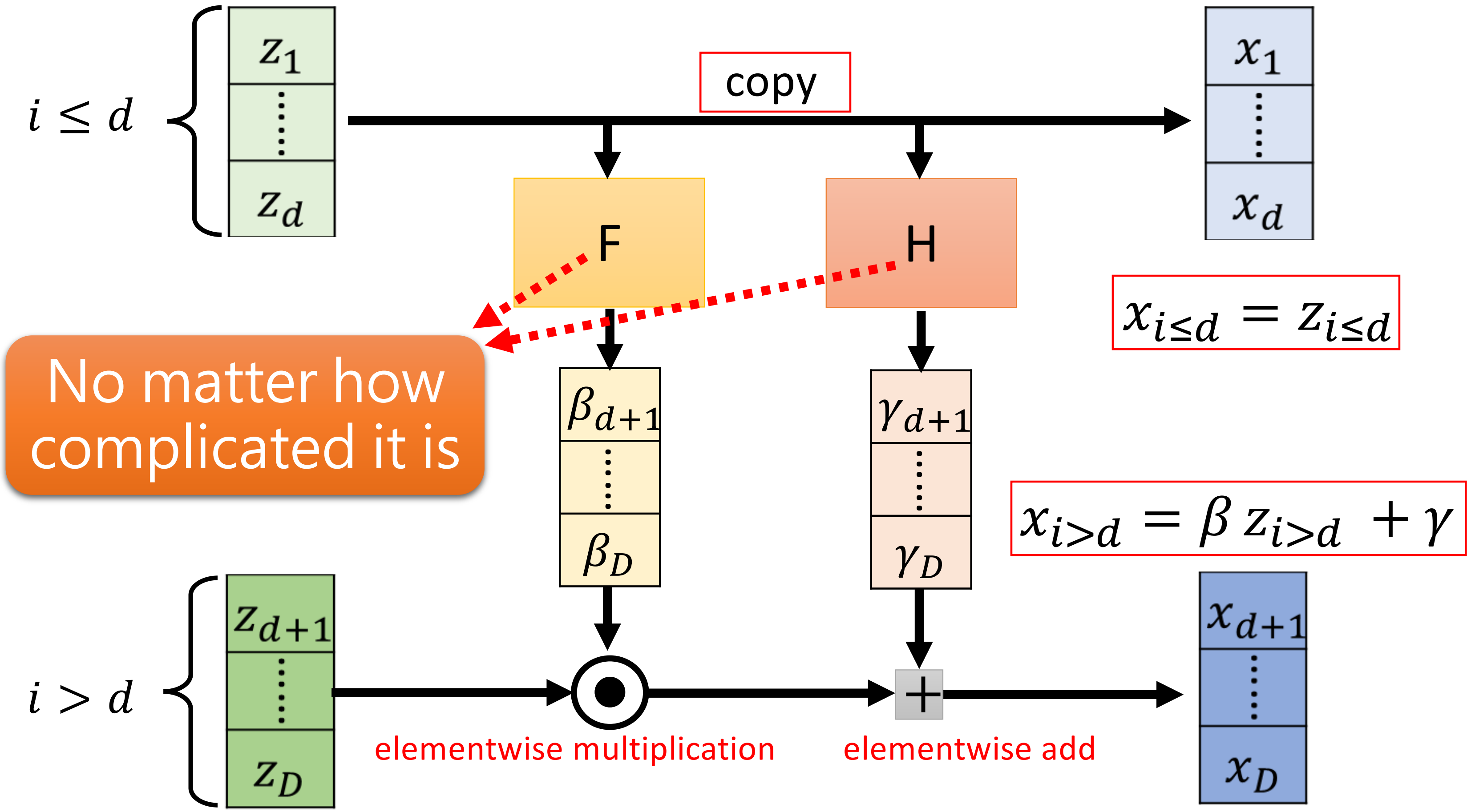
Coupling Layer

NICE
https://arxiv.org/abs/1410.8516
Real NVP
https://arxiv.org/abs/1605.08803

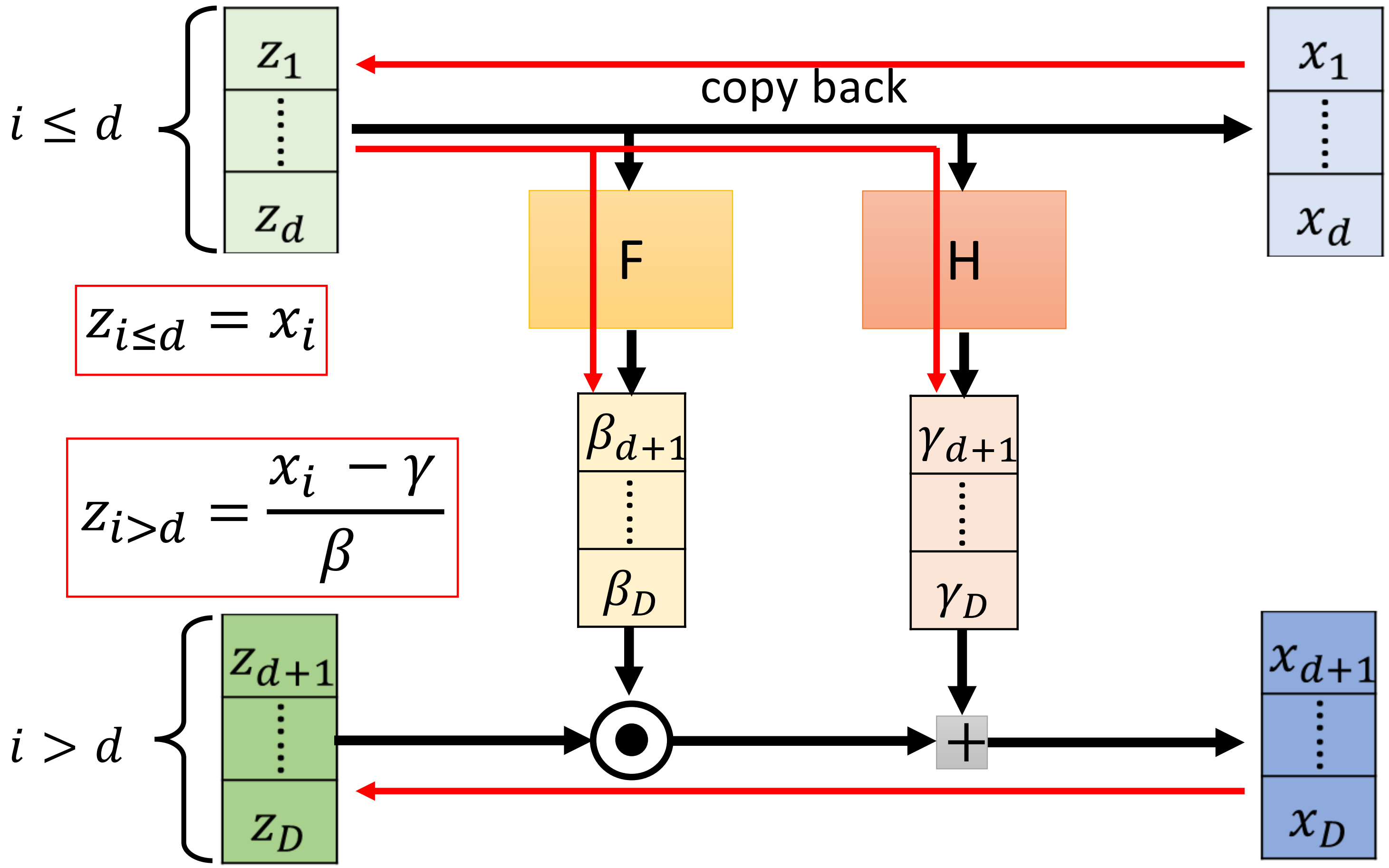


Coupling Layer

NICE
https://arxiv.org/abs/1410.8516
Real NVP
https://arxiv.org/abs/1605.08803

$$z_{i \le d} = x_i$$

$$z_{i>d} = \frac{x_i - \gamma}{\beta}$$

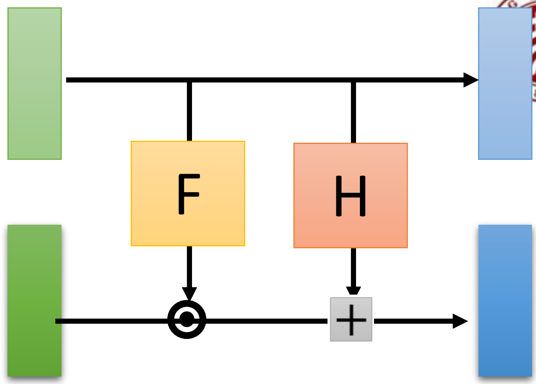


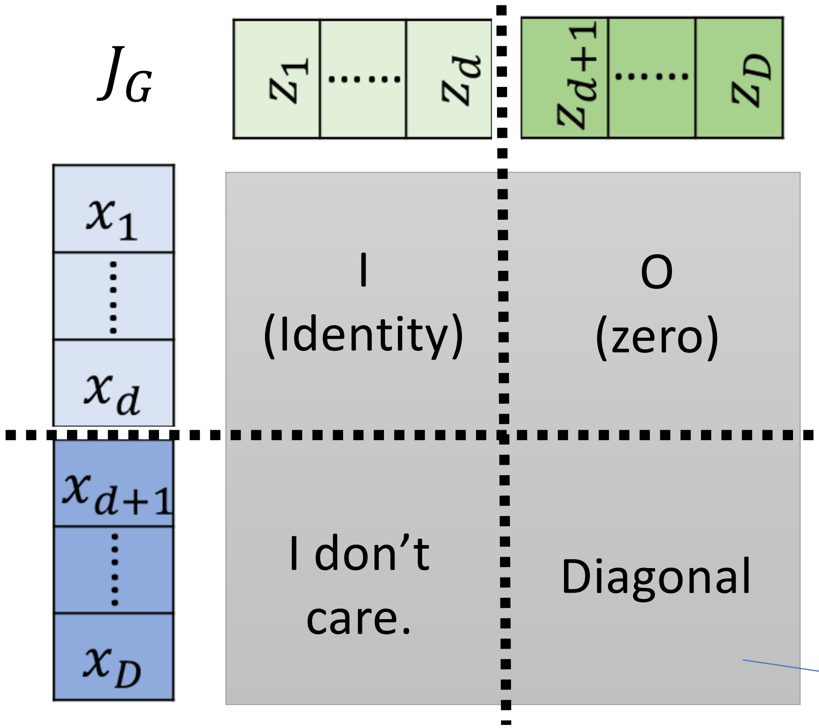
Coupling Layer

- Learning via **maximum likelihood** over the dataset D

$$\max_{\theta} logp(D;\theta) = \sum_{x\in D} log\pi\left(G_\theta^{-1}(x)\right) + log\left|det\left(\underbrace{\frac{\partial G_\theta^{-1}(\boldsymbol{x})}{\partial \boldsymbol{x}}}_{\text{Jacobian}}\right)\right|$$

Coupling Layer





$$det(J_G)$$

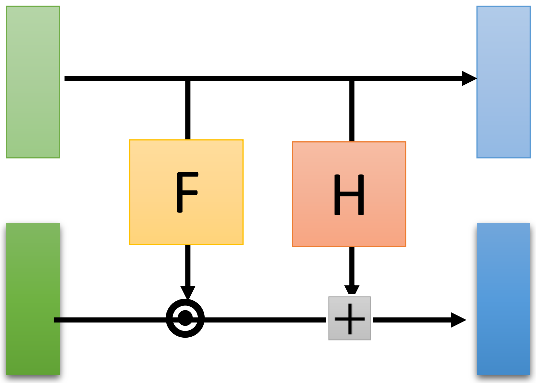
$$= \frac{\partial x_{d+1}}{\partial z_{d+1}} \frac{\partial x_{d+2}}{\partial z_{d+2}} \cdots \frac{\partial x_{\mathrm{D}}}{\partial z_{\mathrm{D}}}$$

$$= \beta_{d+1} \beta_{d+2} \cdots \beta_D$$

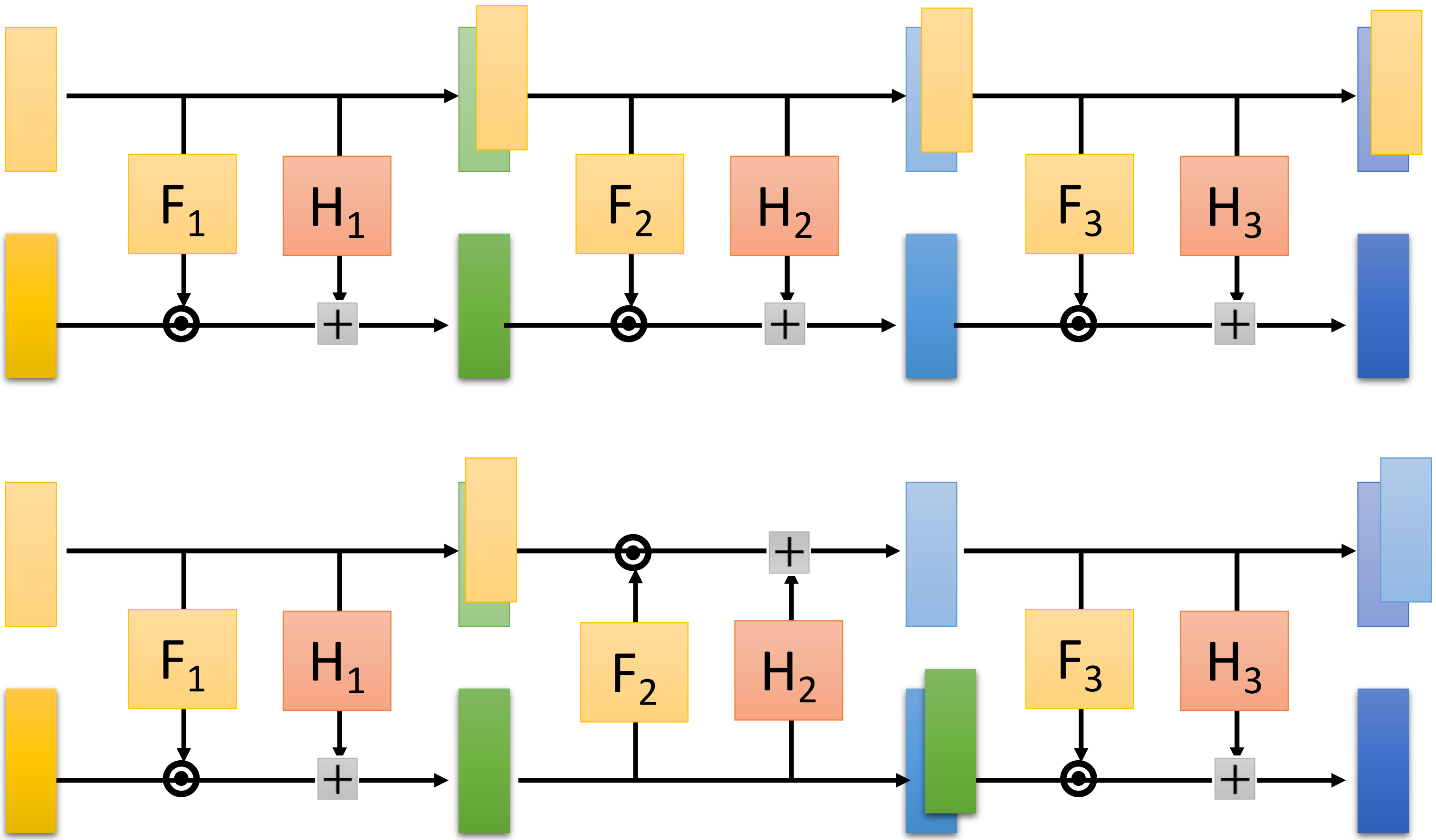
$$x_{i>d} = \beta z_{i>d} + \gamma$$

Coupling Layer

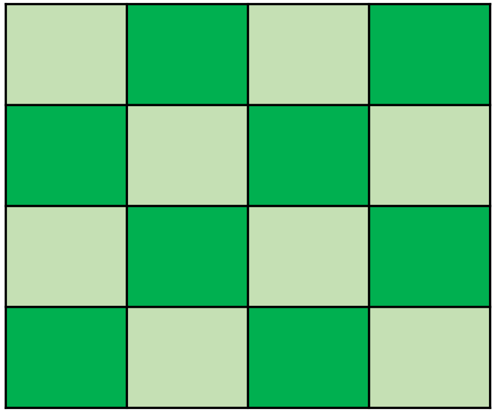
- We can use coupling layer to design invertible function and calculate the determinant of Jacobian efficiently!

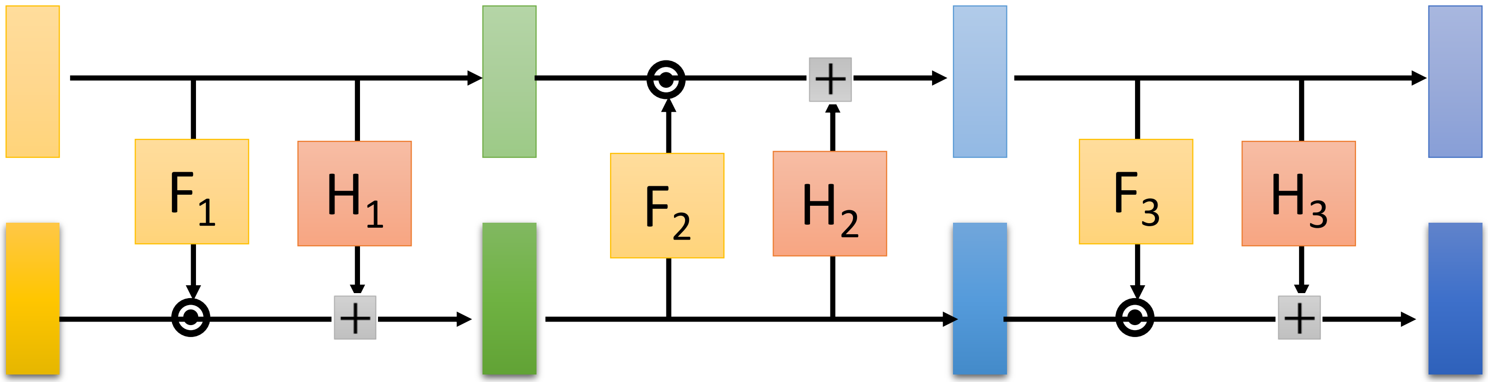


Coupling Layer - Stacking



Coupling Layer





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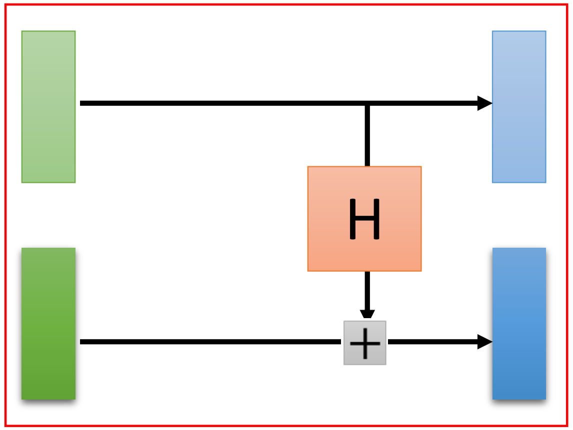
NICE: Nonlinear Independent Components Estimation

- **Additive** coupling layers
  - Partition the variables **z** into two disjoint subsets
  - $x_{1:d} = z_{1:d}$
  - $x_{d+1:n} = z_{d+1:n} + H(z_{1:d})$
  - **Volume preserving transformation** since determinant is 1.



- Additive coupling layers are composed together (with arbitrary partitions of variables in each layer)

- Final layer of NICE applies a rescaling transformation

Dinh et al., 2014. Nonlinear Independent Components Estimation

NICE - Rescaling layers

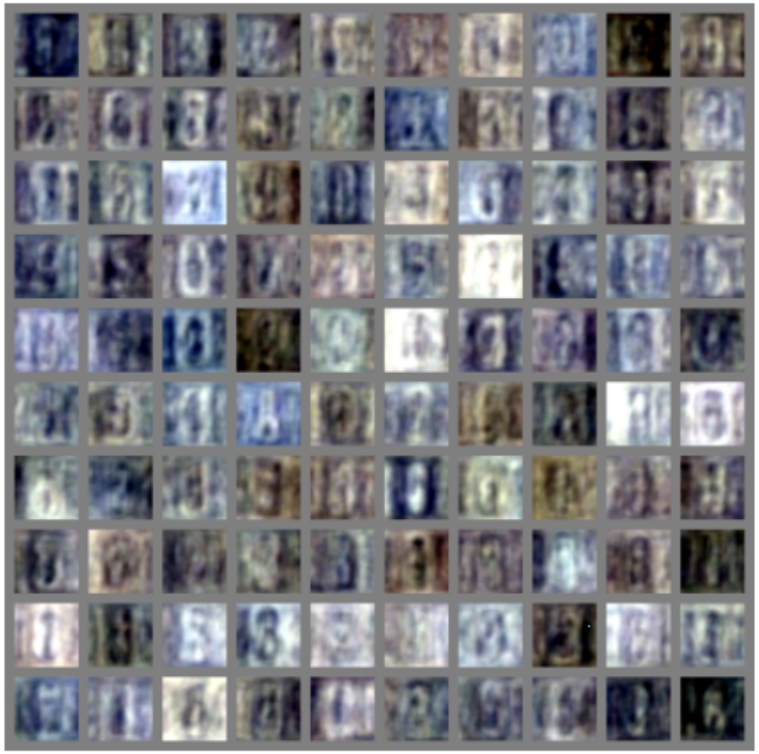
- Rescaling layers
  - Forward:
    - $x_i = s_i z_i$, where $s_i > 0$ is the scaling factor for the i-th dimension.
  - Inverse:
    - $z_i = \mathrm{x_i}/\mathrm{s_i}$
  - Jacobian:
    - $J = diag(\boldsymbol{s})$

Samples generated via NICE

(a) Model trained on MNIST

(b) Model trained on TFD

Samples generated via NICE



(c) Model trained on SVHN

(d) Model trained on CIFAR-10

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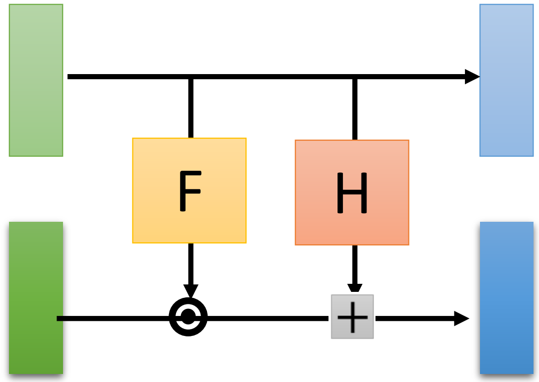
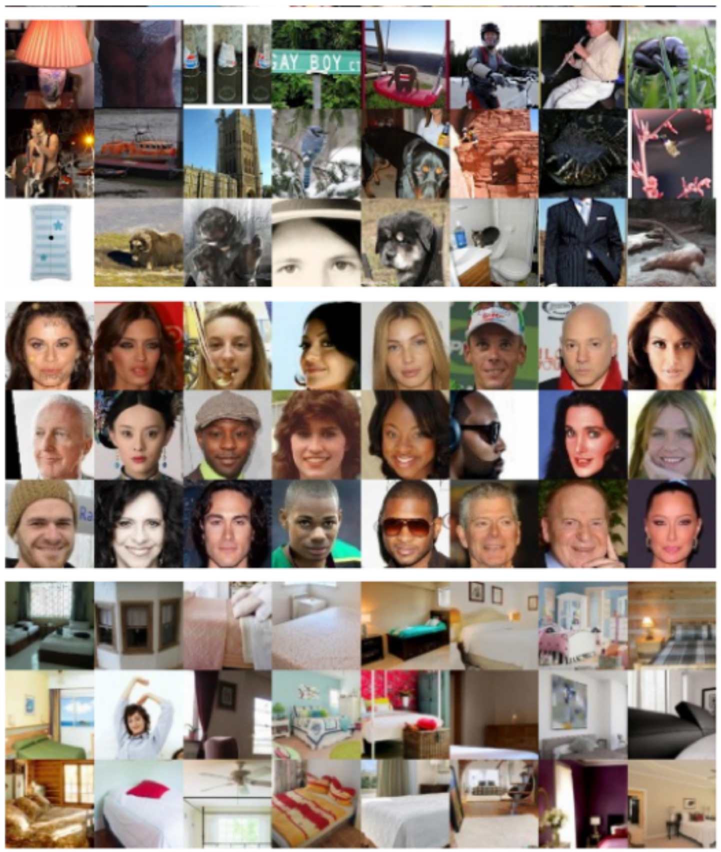
Real NVP

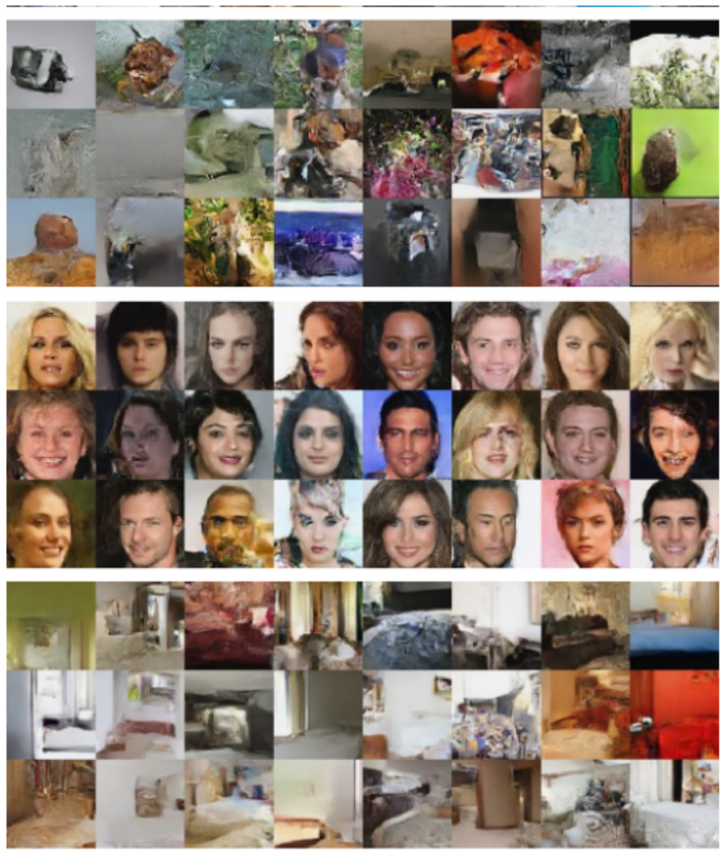


- Coupling layers
  - Partition the variables **z** into two disjoint subsets
  - $x_{1:d} = z_{1:d}$
  - $x_{d+1:n} = z_{d+1:n} \odot F(z_{1:d}) + H(z_{1:d})$
  - **Non-volume preserving transformation** in general since determinant can be less than or greater than 1
- Coupling layers are composed together (with arbitrary partitions of variables in each layer)

Dinh et al., 2017. Density estimation using Real NVP

Samples generated via Real-NVP





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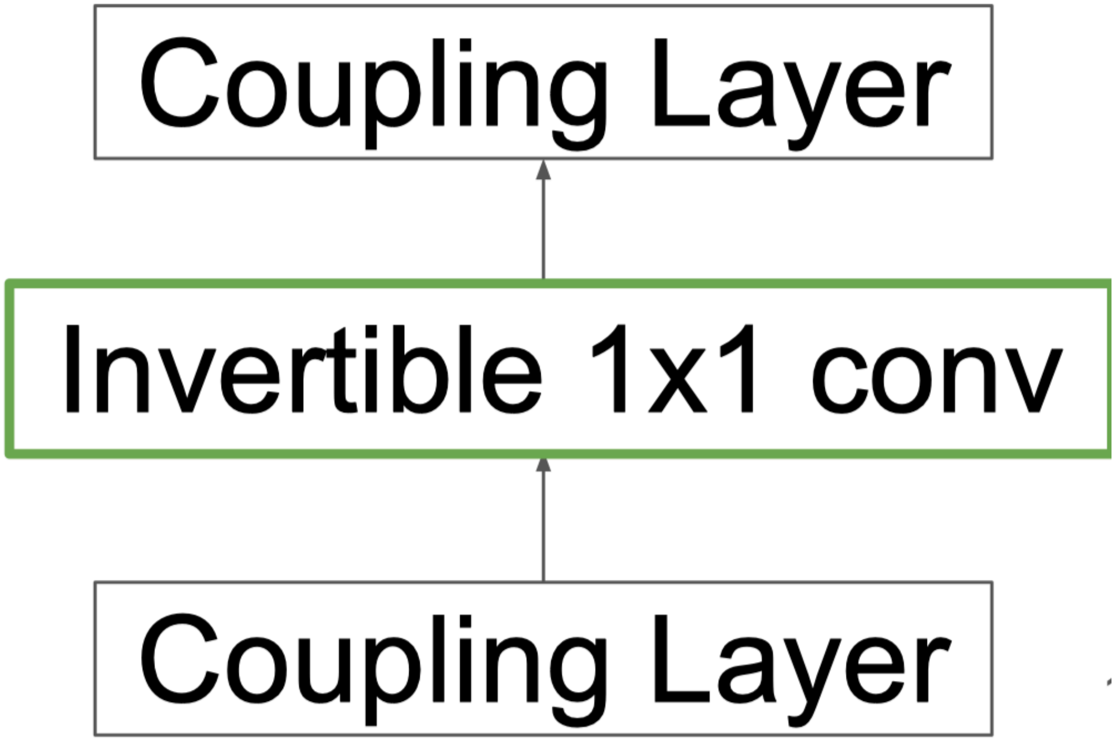
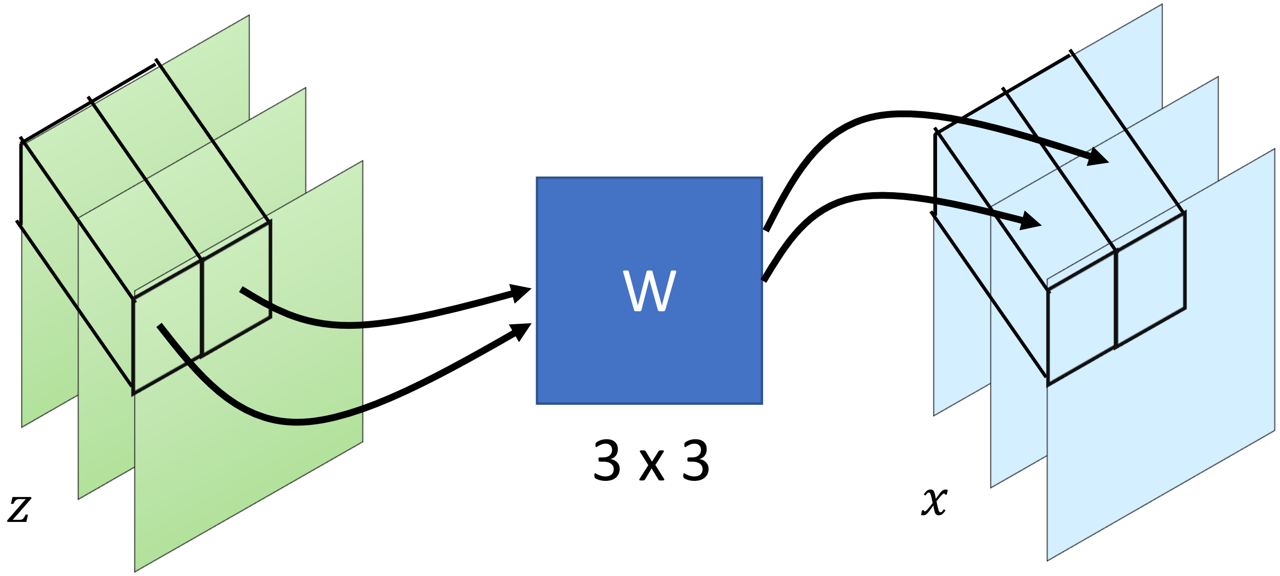
Glow: Generative Flow with Invertible 1×1 Convolutions



Kingma et al. Glow: Generative Flow with Invertible 1x1 Convolutions

1x1 Convolution

GLOW
https://arxiv.org/abs/1807.03039



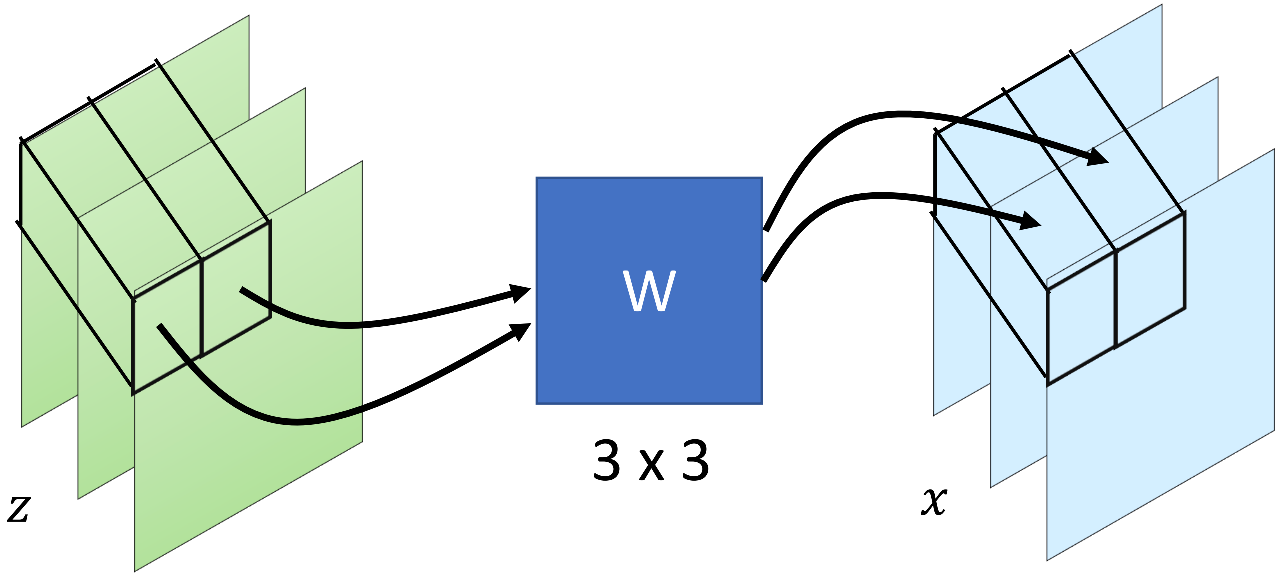
W can shuffle the channels.

If W is invertible, it is easy to compute W^{-1}.

$$\begin{bmatrix}3\\1\\2\end{bmatrix} = \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}$$

1x1 Convolution

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$



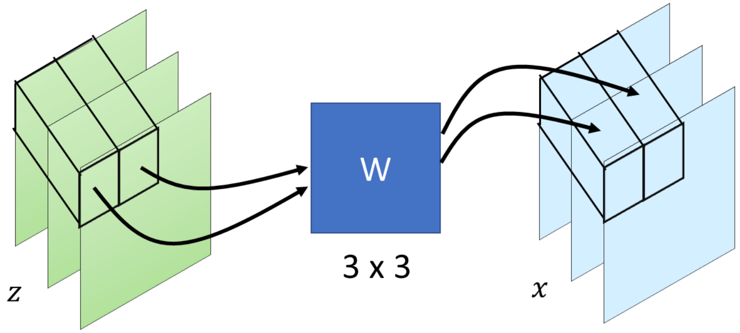
$$x = f(z) = Wz$$

$$J_f = \begin{bmatrix} \partial x_1/\partial z_1 & \partial x_1/\partial z_2 & \partial x_1/\partial z_3 \\ \partial x_2/\partial z_1 & \partial x_2/\partial z_2 & \partial x_2/\partial z_3 \\ \partial x_3/\partial z_1 & \partial x_3/\partial z_2 & \partial x_3/\partial z_3 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} = W$$

1x1 Convolution

$$\boxed{(det(W))^{d\times d}}$$

If W is 3x3, computing $det(W)$ is easy.



$d\times d$
positions
(pixels)

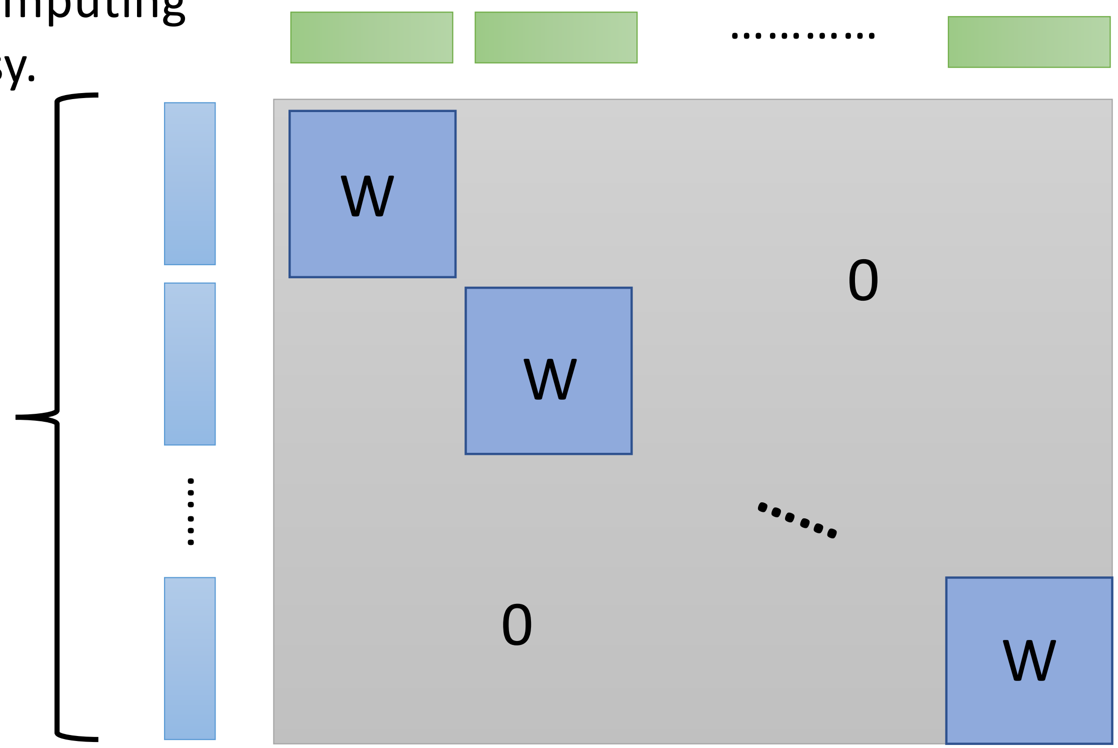


Image results: Glow

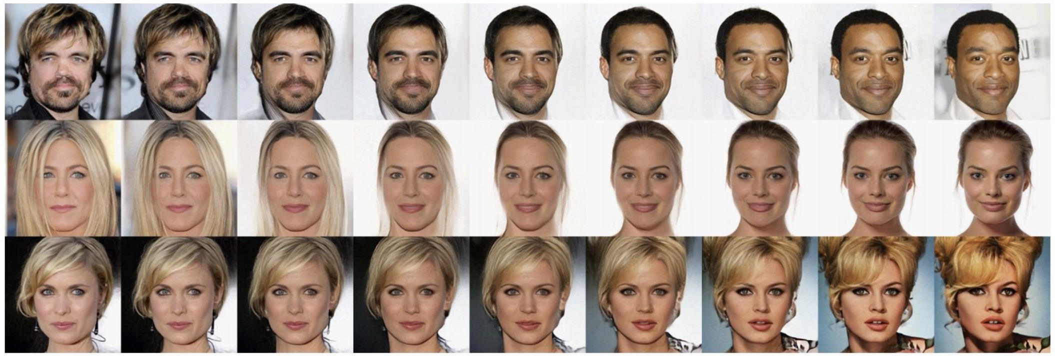
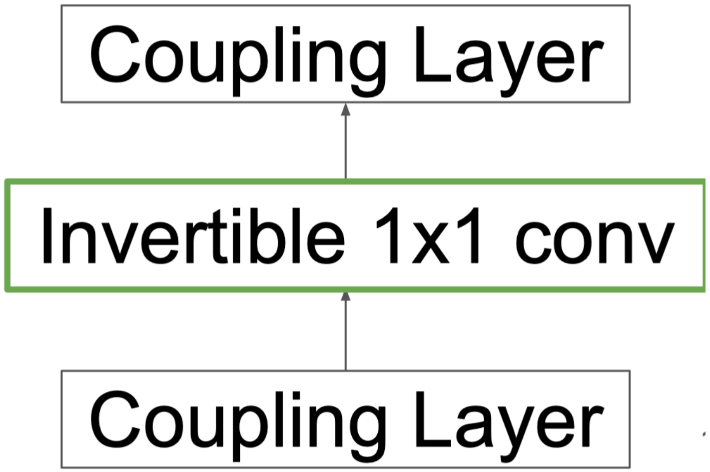


Figure 5: Linear interpolation in latent space between real images



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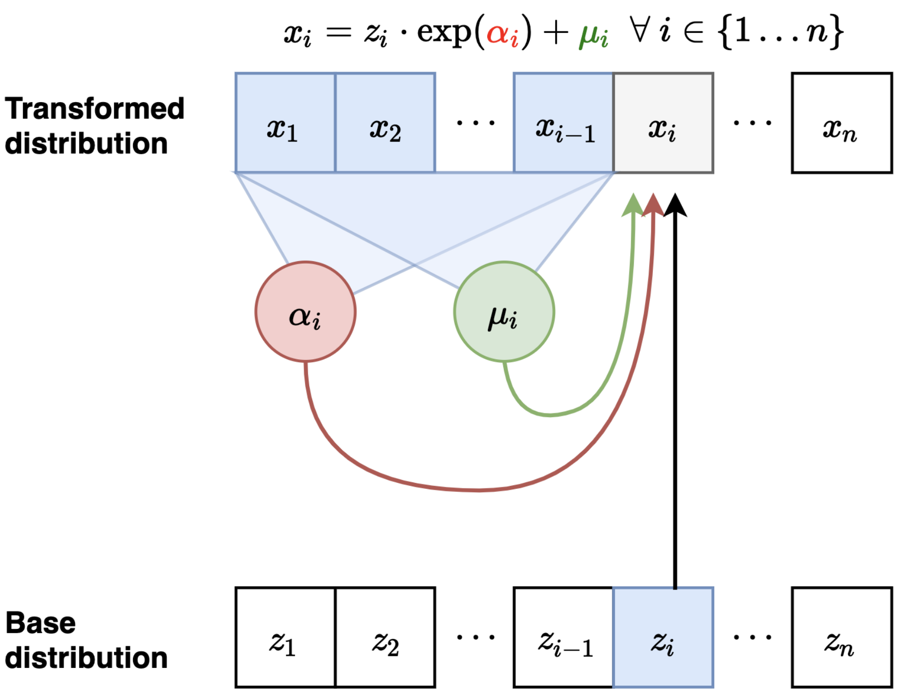
Autoregressive models as flow models

- Consider a Gausian autoregressive model:
  - $p(\boldsymbol{x}) = \prod_{i=1}^{\mathrm{n}} p(x_i|\boldsymbol{x}_{<\boldsymbol{i}})$
  - Such that $p(x_i|\boldsymbol{x}_{<\boldsymbol{i}}) = N(\mu_i(x_1, \dots, x_{i-1}), \exp\left(\alpha_i(x_1, \dots, x_{i-1})\right)^2)$, μ_i, α_i are neural networks.
- Sampler for this model:
  - Sample $z_i \sim N(0,1)$
  - Let $x_i = \exp(\alpha_i)\, z_i + \mu_i$ ← look like coupling layer ~~
- **Flow interpretation:** transform **z** to **x** via invertible transformation (parameterized by μ_i, α_i)

Masked Autoregressive Flow (MAF)

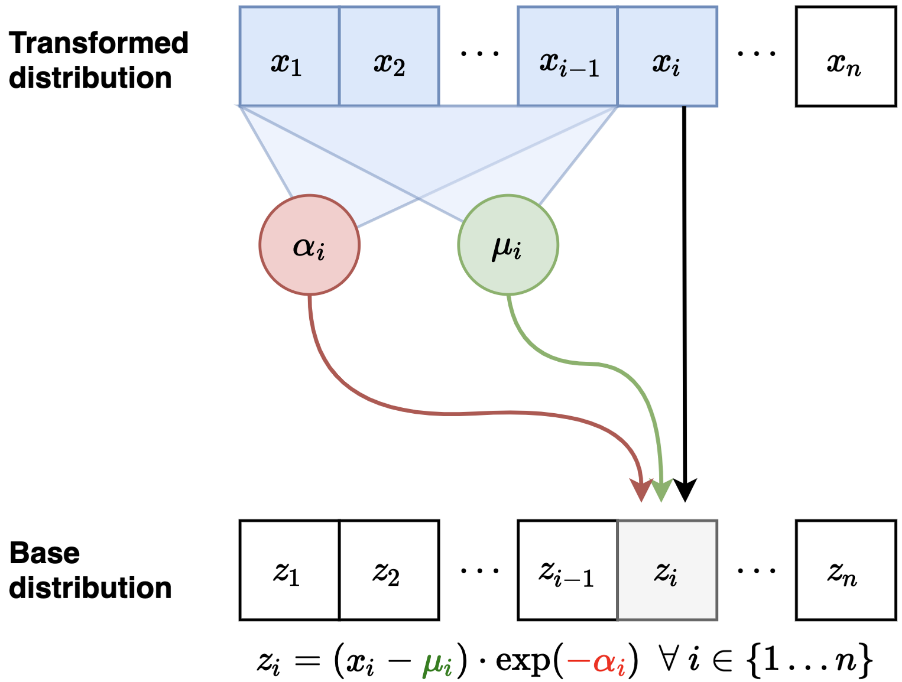


- Forward: **(z** to **x)**
  - $x_i = z_i \exp(\alpha_i) + \mu_i$
  - Then calculate α_{i+1}, μ_{i+1}
- Sampling is sequential and slow (like autoregressive)

Figure adapted from Eric Jang's blog

Papamakarios et al. Masked Autoregressive Flow for Density Estimation

Masked Autoregressive Flow (MAF)



- Inverse **(x** to **z)**
  - $z_i = (x_i - \mu_i)\exp(-\alpha_i)$
- can be done in parallel.

- Jacobian is lower diagonal, hence determinant can be computed efficiently
- Likelihood evaluation is easy and parallelizable

Figure adapted from Eric Jang's blog

- $\max_{\theta} logp(D;\theta) = \sum_{x\in D} log\pi\left(G_\theta^{-1}(x)\right) + log\left|det\left(\frac{\partial G_\theta^{-1}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)\right|$

- MAF can calculate $G_\theta^{-1}(x)$ parallel.

- MAF: Fast likelihood evaluation (parallel), slow sampling(sequential)

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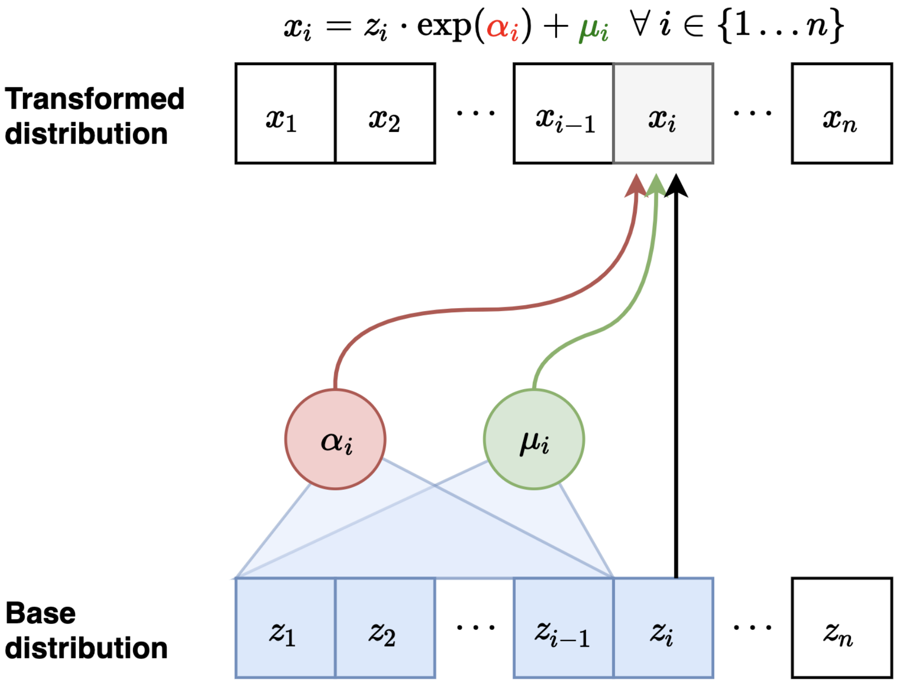
Inverse Autoregressive Flow (IAF)



- Forward: **(z to x)**
  - $x_i = z_i \exp(\alpha_i) + \mu_i$
  - parallel
- Inverse **(x to z)**
  - $z_i = (x_i - \mu_i)\exp(-\alpha_i)$
  - Then compute μ_i, α_i
  - sequential

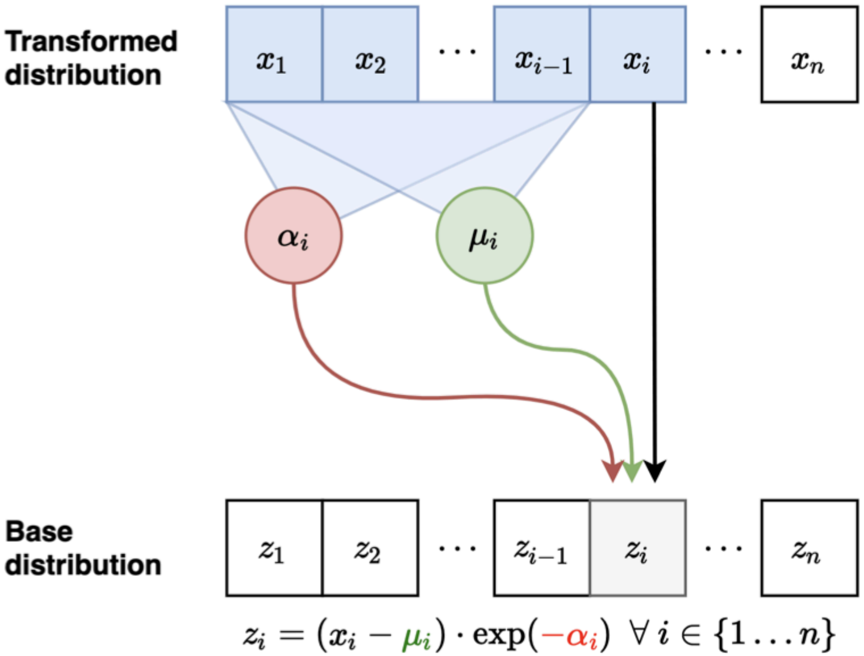
Figure adapted from Eric Jang's blog

Kingma et al. Improving Variational Inference with Inverse Autoregressive Flow

Inverse Autoregressive Flow (IAF)

- Fast to sample (parallel)
- Slow to evaluate likelihoods of data points during training (sequential)
- Fast to evaluate likelihoods of a generated point (we only need to cache $z_1, z_2, \dots, z_n$)

IAF is inverse of MAF



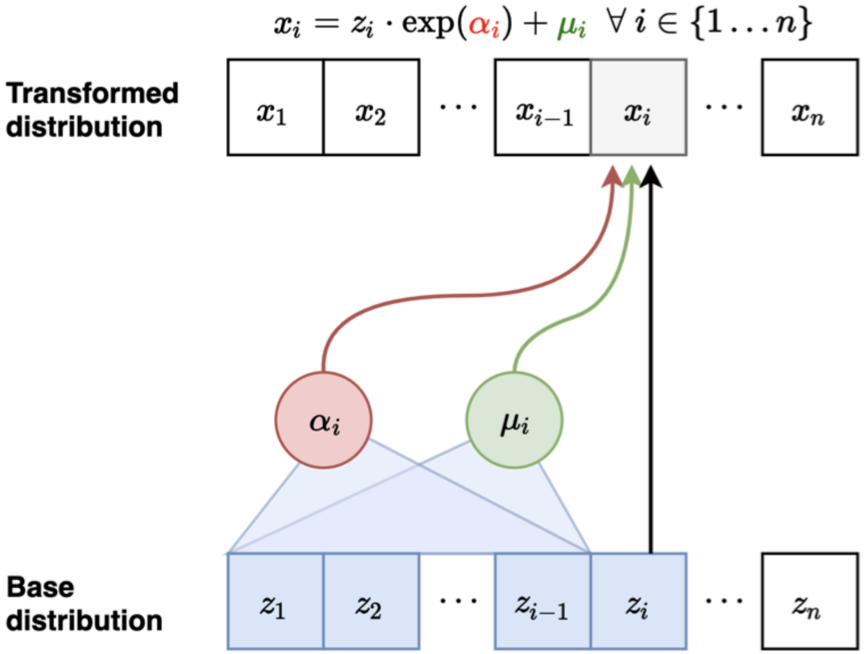


Figure: Inverse pass of MAF (**left**) vs. Forward pass of IAF (**right**)

IAF vs. MAF

- Computational tradeoffs
  - MAF: Fast likelihood evaluation, slow sampling
  - IAF: Fast sampling, slow likelihood evaluation
- MAF more suited for training based on MLE, density estimation
- IAF more suited for real-time generation
- Can we get the best of both worlds?

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Parallel Wavenet

MAF: $x \mapsto z$ parallel
IAF: $z \mapsto x$ parallel

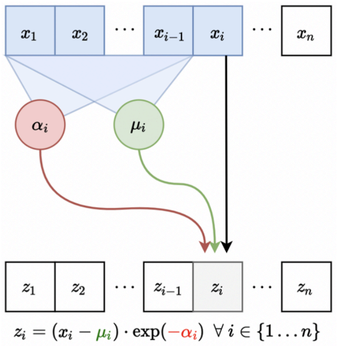
- Two part training with a teacher (MAF) and student model (IAF)
- Teacher can be efficiently trained via MLE.
- Once teacher is trained, initialize a student model parameterized by IAF. Student model cannot efficiently evaluate density for external data points but allows for efficient sampling
- **Key observation:** IAF can also efficiently evaluate densities of its own generations (via caching the noise variates $z_1, z_2, \dots, z_n$)

Oord et al. Parallel WaveNet: Fast High-Fidelity Speech Synthesis. 2017

Parallel Wavenet

MAF: $x \mapsto z$ parallel

IAF: $z \mapsto x$ parallel



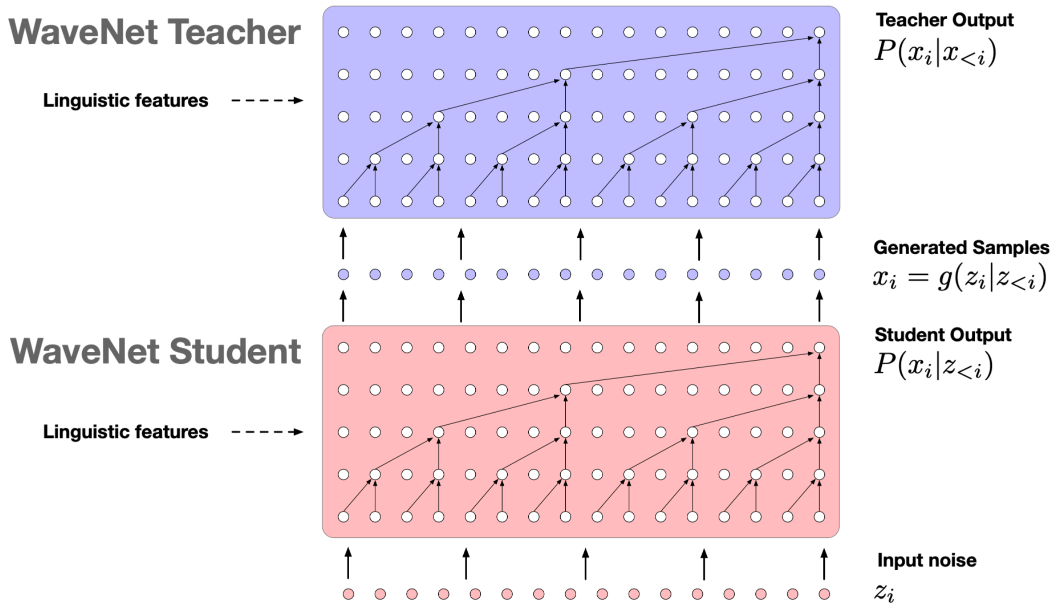
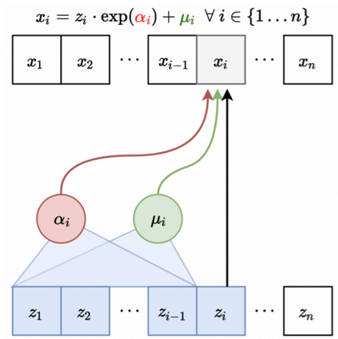


Figure 2: **Overview of Probability Density Distillation**. A pre-trained WaveNet teacher is used to score the samples $\boldsymbol{x}$ output by the student. The student is trained to minimise the KL-divergence between its distribution and that of the teacher by maximising the log-likelihood of its samples under the teacher and maximising its own entropy at the same time.



Oord et al. Parallel WaveNet: Fast High-Fidelity Speech Synthesis. 2017

Parallel Wavenet

MAF: $x \mapsto z$ parallel
IAF: $z \mapsto x$ parallel

- **Probability density distillation**: Student distribution is trained to minimize the KL divergence between student (s) and teacher (t)

$$D_{KL}(s,t) = E_{\boldsymbol{x}\sim s}[\log(\mathrm{s}(\boldsymbol{x})) - \log(t(\boldsymbol{x}))]$$

- Evaluating and optimizing Monte Carlo estimates of this objective requires:
  - Samples **x** from student model (IAF)
  - Density of **x** assigned by student model (IAF)
  - Density of **x** assigned by teacher model (MAF)
- All operations above can be implemented efficiently!

Parallel Wavenet: Overall algorithm

- Training
  - Step 1: Train teacher model (MAF) via MLE
  - Step 2: Train student model (IAF) to minimize KL divergence with teacher
- Test-time: Use student model for testing
- Improves sampling efficiency over original Wavenet (vanilla autoregressive model) by 1000x!
- Useful in speech synthesis

- Coupling layer based normalizing flow models
  - Coupling layer
  - NICE add only
  - Real NVP add+mul
  - Glow conv 1x1
- Autoregressive models as flow models
  - MAF fast train, slow test
  - IAF fast test, slow train
  - Parallel Wavenet fast train, fast test

Summary of Normalizing Flow Models

- Transform simple distributions into more complex distributions via change of variables
- Jacobian of transformations should have tractable determinant for efficient learning and density estimation
- Computational tradeoffs in evaluating forward and inverse transformations

Thanks